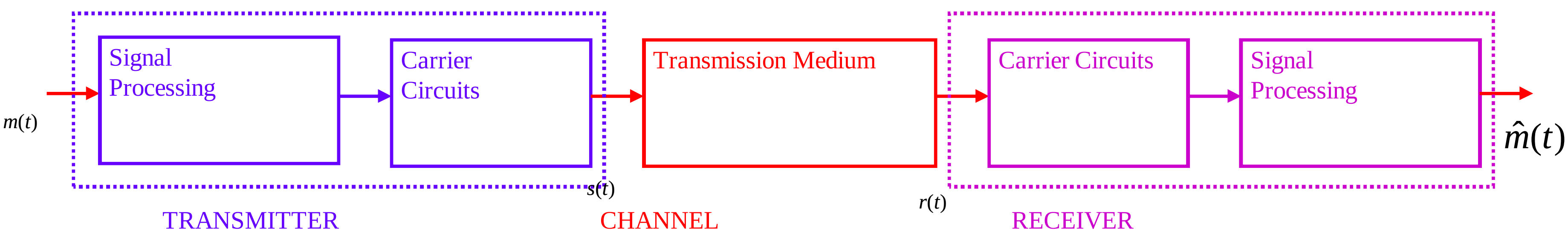
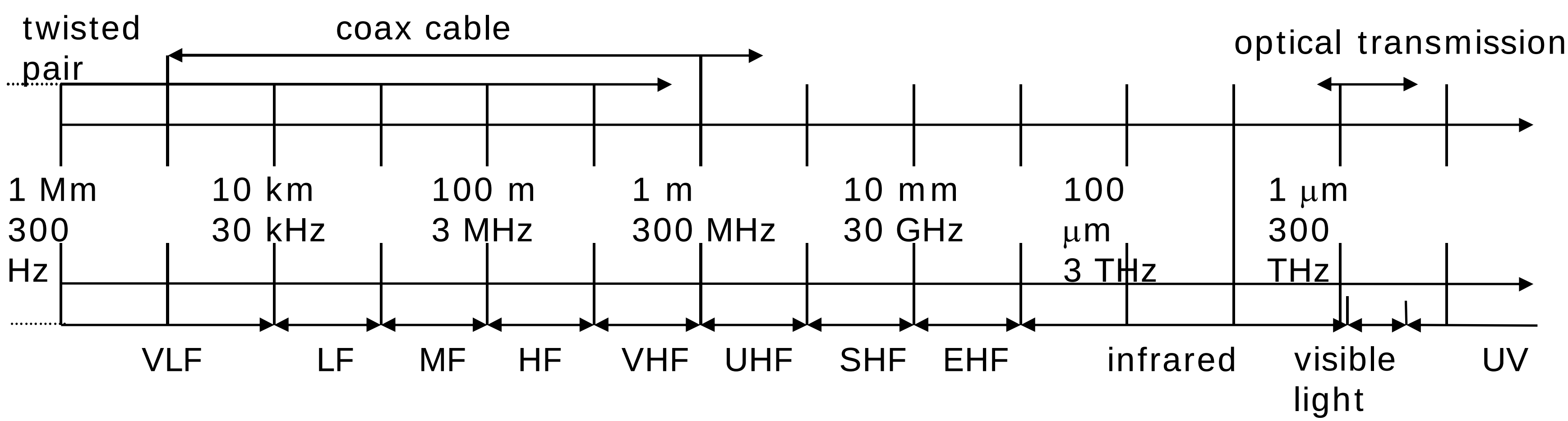
UNIT-I

Amplitude Modulation System

- Elements of a communication system (cont)
  - Basic components
    - Transmitter
      - Convert Source (information) to signals
      - Send converted signals to the channel (by antenna if applicable)
    - Channel
      - Wireless: atmosphere (free space)
      - Wired: coaxial cables, twisted wires, optical fibre
    - Receiver
      - Reconvert received signals to original information
      - Output the original information



- Elements of a communication system (cont)
  - ➢Frequencies for communication



- VLF = Very Low Frequency
- LF = Low Frequency
- MF = Medium Frequency
- HF = High Frequency
- VHF = Very High Frequency

UHF = Ultra High Frequency

SHF = Super High Frequency

EHF = Extra High Frequency

UV = Ultraviolet Light

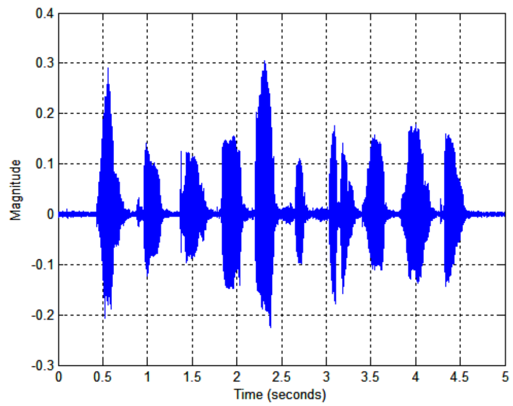
- Frequency and wave length:

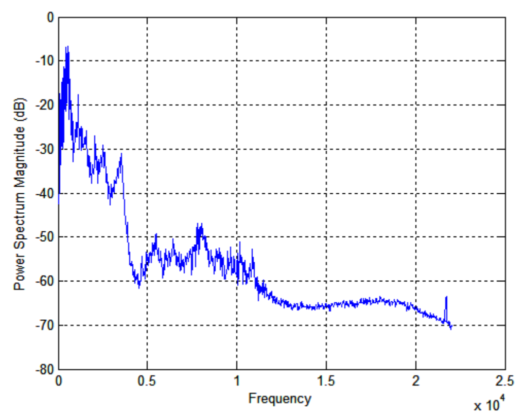
  $\lambda = c/f$

  wave length λ, speed of light $c \cong 3 \times 10^8$ m/s, frequency f

Baseband vs Passband Transmission

- Baseband signals:
  - Voice (0-4kHz)
  - TV (0-6 MHz)
- A signal may be sent in its baseband format when a dedicated wired channel is available.
- Otherwise, it must be converted to passband.





Modulation: What and Why?

- The process of shifting the baseband signal to passband range is called *Modulation*.
- The process of shifting the passband signal to baseband frequency range is called *Demodulation*.
- Reasons for modulation:
  - Simultaneous transmission of several signals
  - Practical Design of Antennas
  - Exchange of power and bandwidth

Types of (Carrier) Modulation

- In modulation, one characteristic of a signal (generally a sinusoidal wave) known as the *carrier* is changed based on the information signal that we wish to transmit (*modulating signal*).
- That could be the amplitude, phase, or frequency, which result in Amplitude modulation (AM), Phase modulation (PM), or Frequency modulation (FM). The last two are combined as Angle Modulation

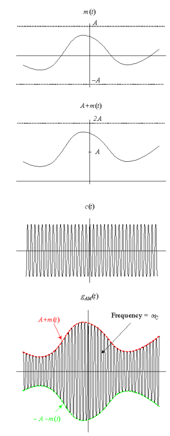
Types of Amplitude Modulation (AM)

- **Double Sideband with carrier (we will call it AM)**: This is the most widely used type of AM modulation. In fact, all radio channels in the AM band use this type of modulation.
- **Double Sideband Suppressed Carrier (DSBSC):** This is the same as the AM modulation above but without the carrier.
- **Single Sideband (SSB):** In this modulation, only half of the signal of the DSBSC is used.
- **Vestigial Sideband (VSB):** This is a modification of the SSB to ease the generation and reception of the signal.

Definition of AM

- Shift $m(t)$ by some DC value "A" such that $A+m(t) \geq 0$. Or $A \geq m_{peak}$

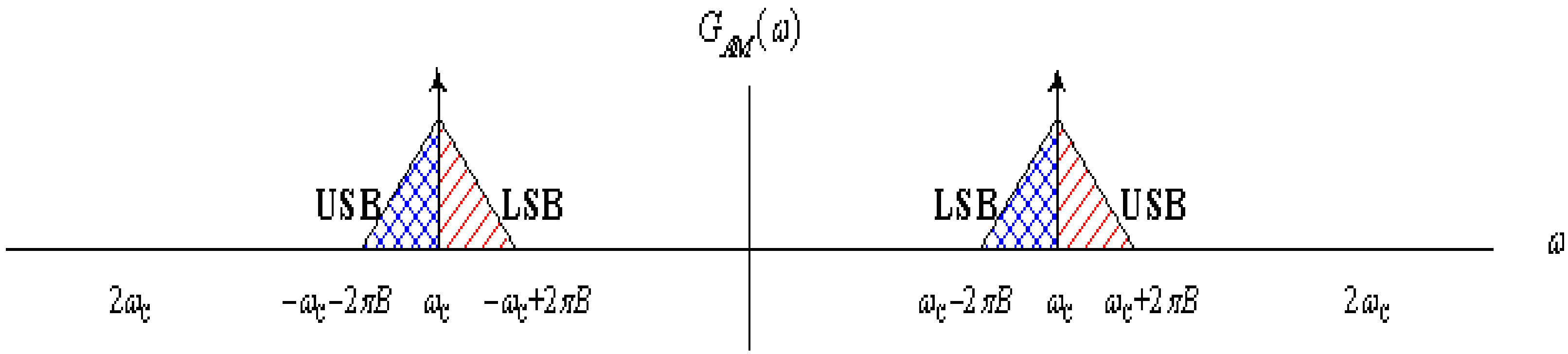
$$g_{AM}(t) = [A + m(t)]\cos(\omega_C t)$$
$$= A\cos(\omega_C t) + m(t)\cos(\omega_C t)$$



- Called DSBWC. Here will refer to it as Full AM, or simply AM
- Modulation index $\mu = m_p / A$.
- $0 \leq \mu \leq 1$

Spectrum of AM

$$g_{AM}(t) \Leftrightarrow \pi A\left[\delta(\omega-\omega_C)+\delta(\omega+\omega_C)\right]+\frac{1}{2}\left[M(\omega-\omega_C)+M(\omega+\omega_C)\right]$$

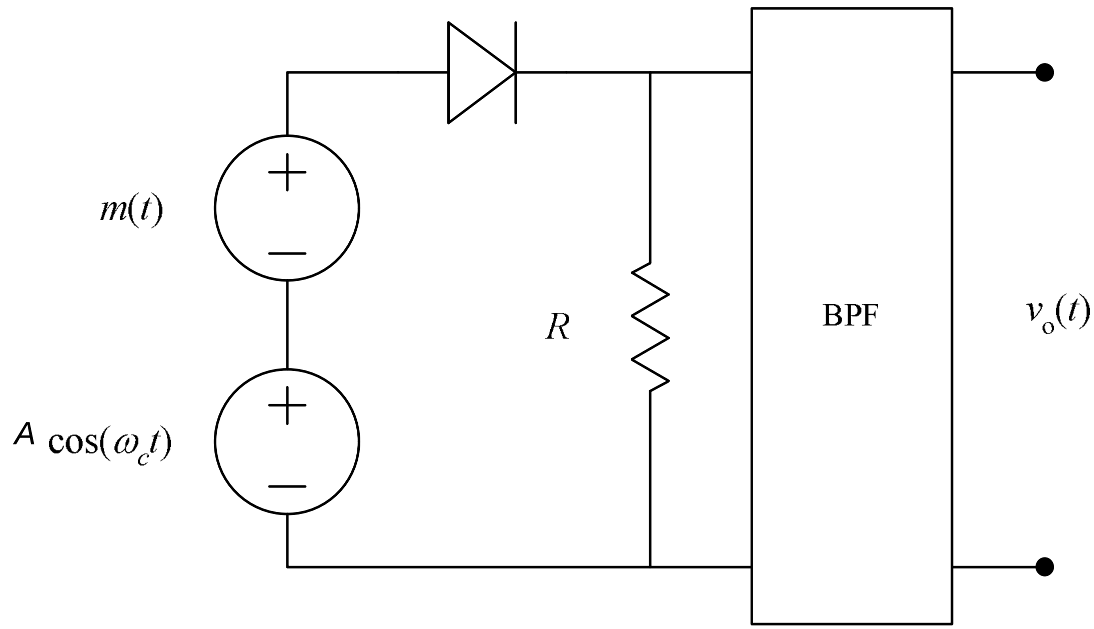


Generation of AM

- AM signals can be generated by any DSBSC modulator, by using $A+m(t)$ as input instead of $m(t)$.
- In fact, the presence of the carrier term can make it even simpler. We can use it for switching instead of generating a local carrier.
- The switching action can be made by a single diode instead of a diode bridge.

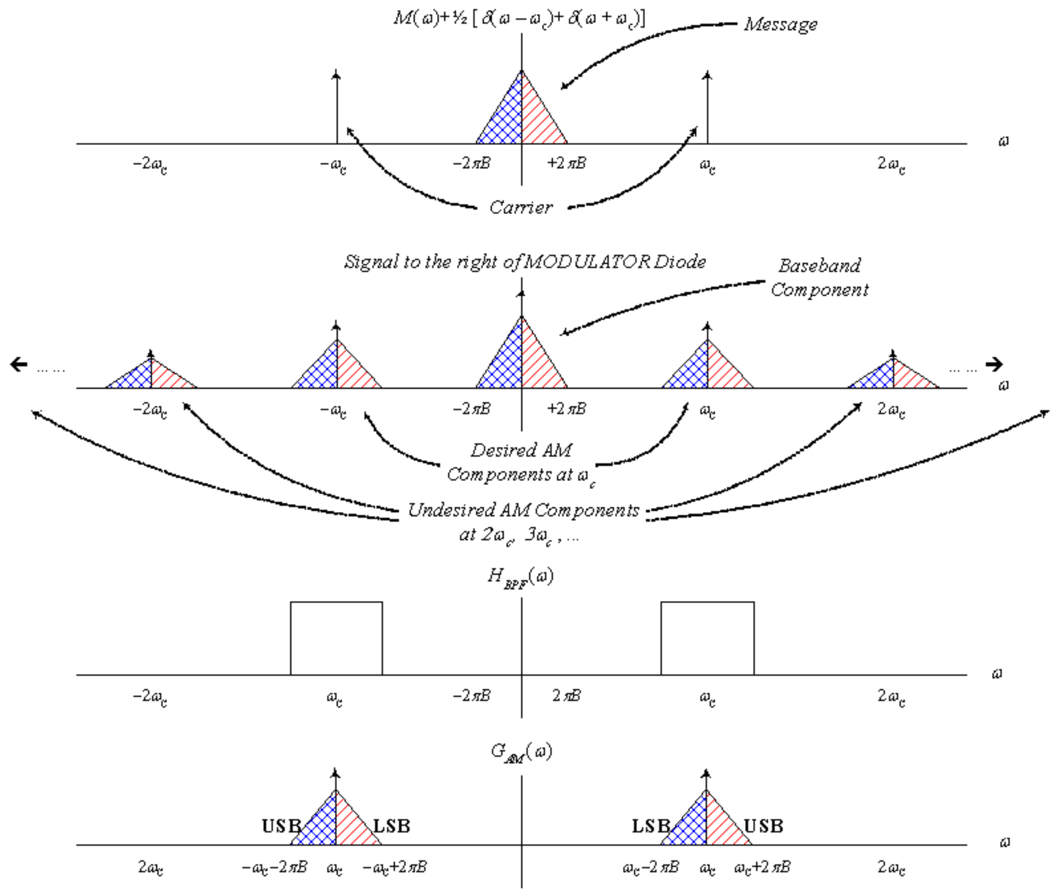
AM Generator

- $A \gg m(t)$
  (to ensure switching at every period).



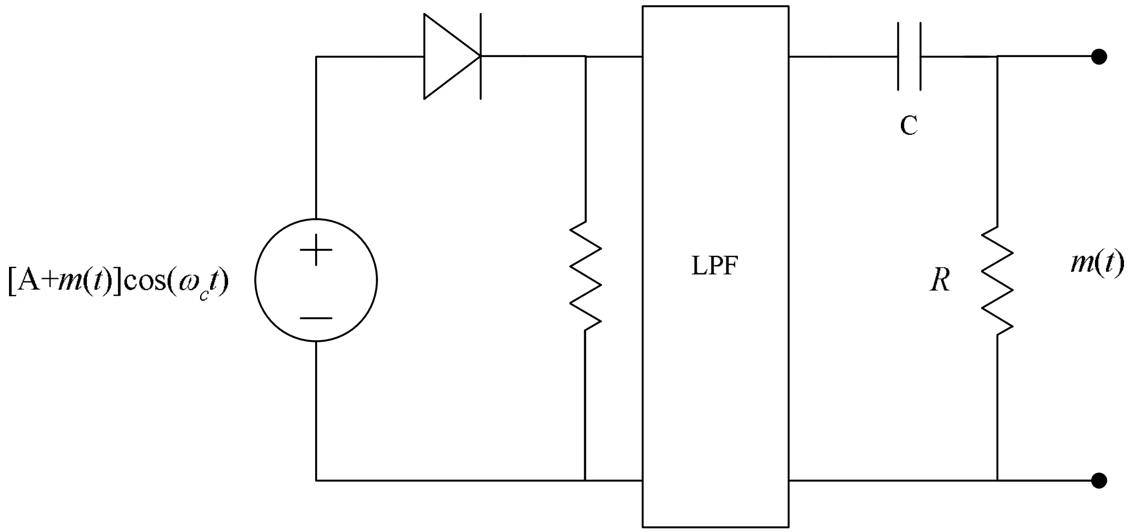
- $v_R = [\cos\omega_c t + m(t)][1/2 + 2/\pi(\cos\omega_c t - 1/3\cos 3\omega_c t + \ldots)]$
  $= (1/2)\cos\omega_c t + (2/\pi) m(t) \cos\omega_c t$ + other terms (suppressed by BPF)
- $v_o(t) = (1/2)\cos\omega_c t + (2/\pi) m(t) \cos\omega_c t$

AM Modulation Process (Frequency)



AM Demodulation: Rectifier Detector

- Because of the presence of a carrier term in the received signal, switching can be performed in the same way we did in the modulator.



Rectifier Detector: Time Domain

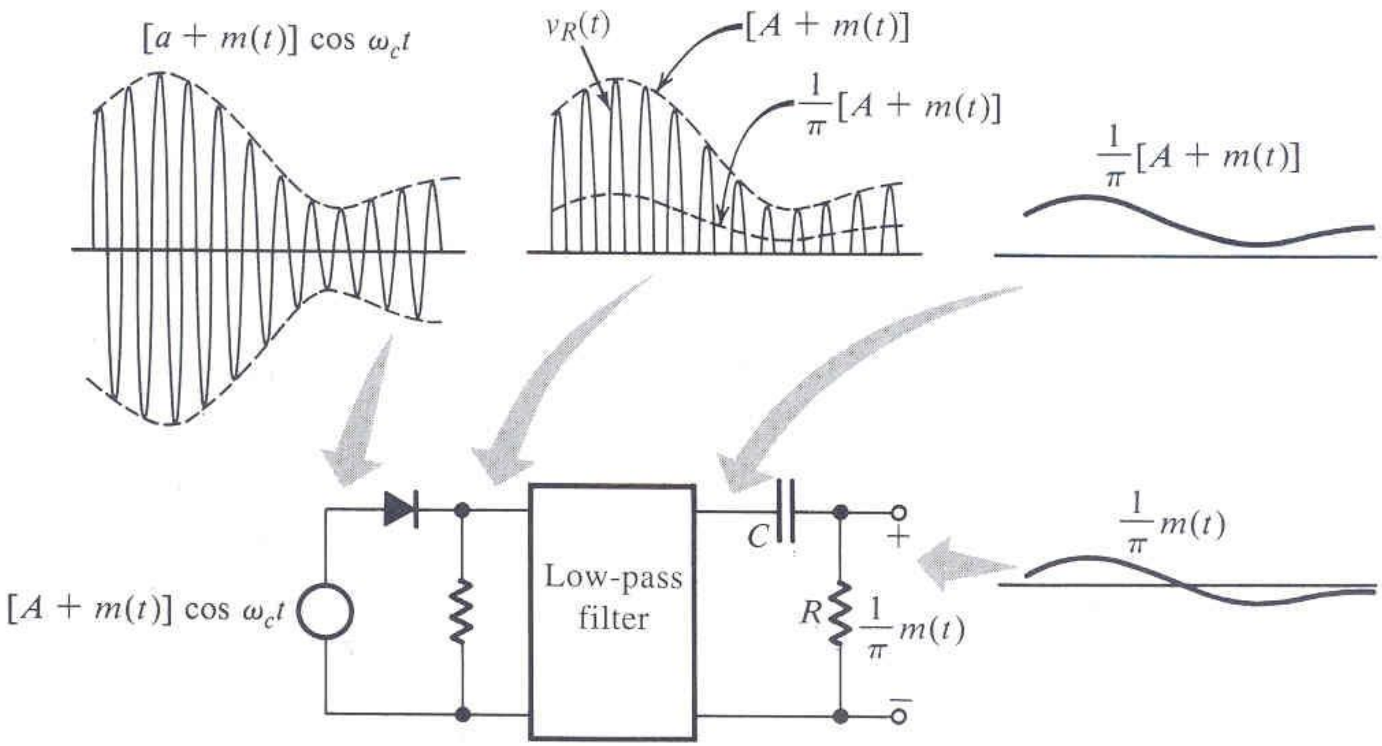
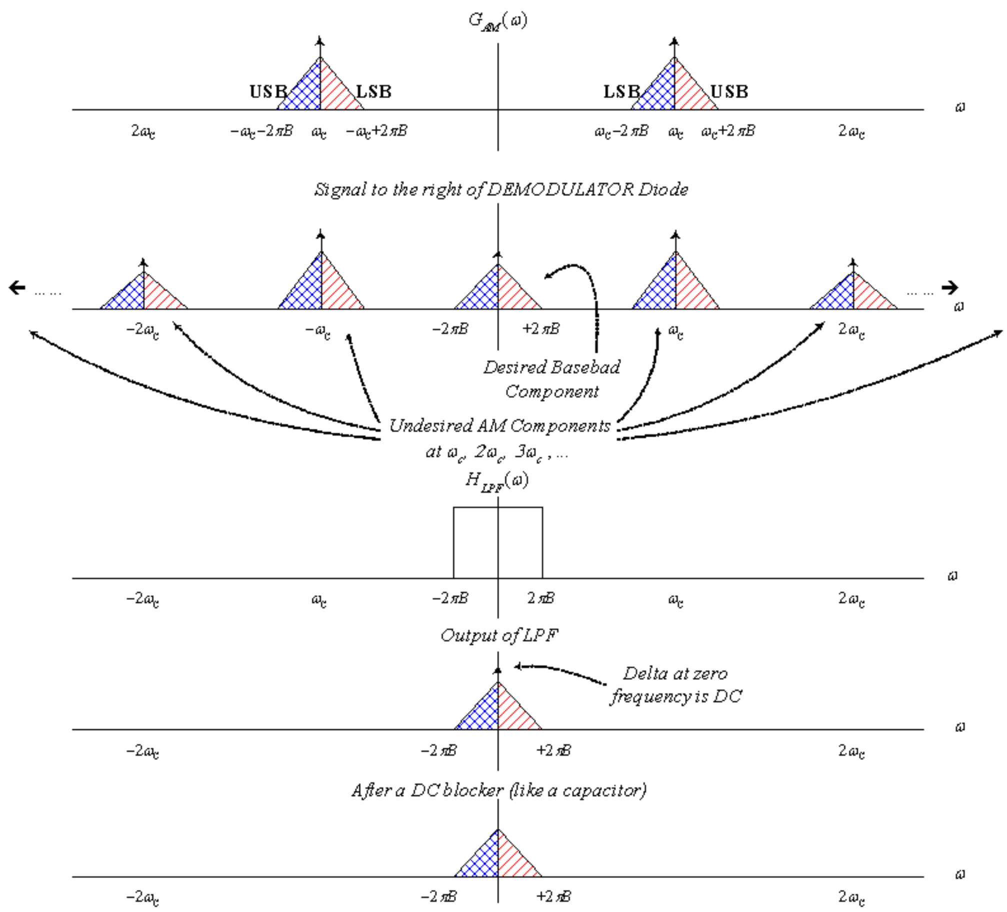
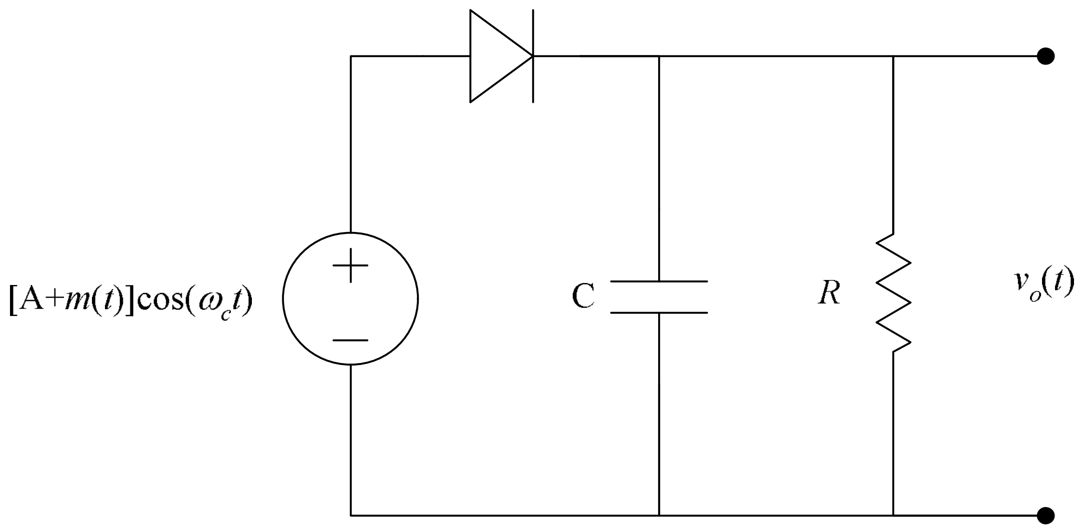


Figure 4.11 Rectifier detector for AM.

Rectifier Detector (Frequency Domain)

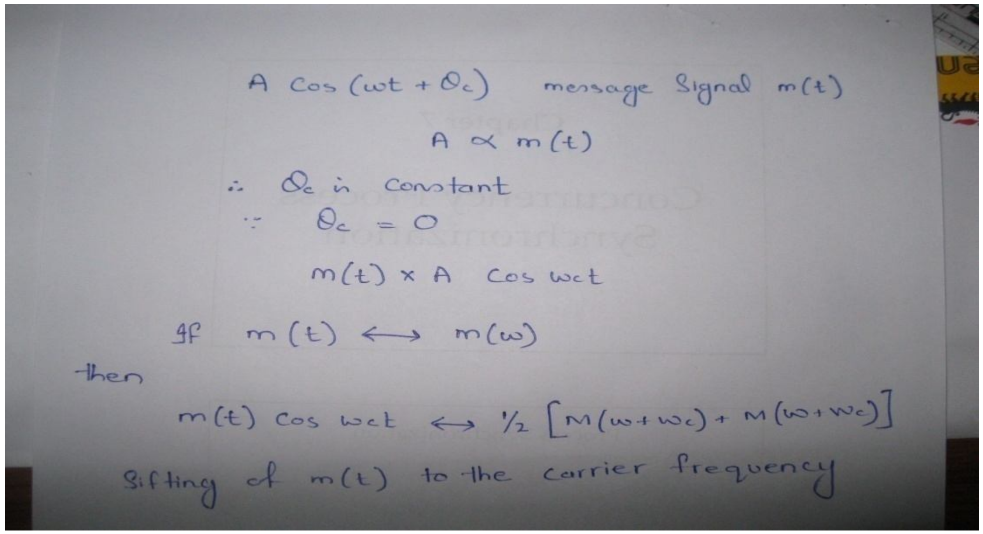


Envelope Detector



- When D is forward-biased, the capacitor charges and follows input.
- When D is reverse-biased, the capacitor discharges through *R*.

Double Sideband Suppressed Carrier (DSBSC)

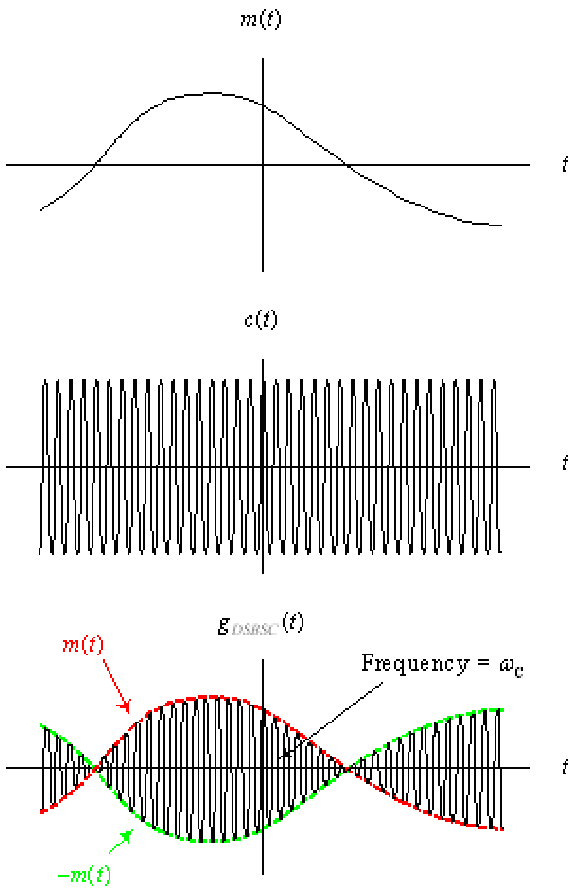


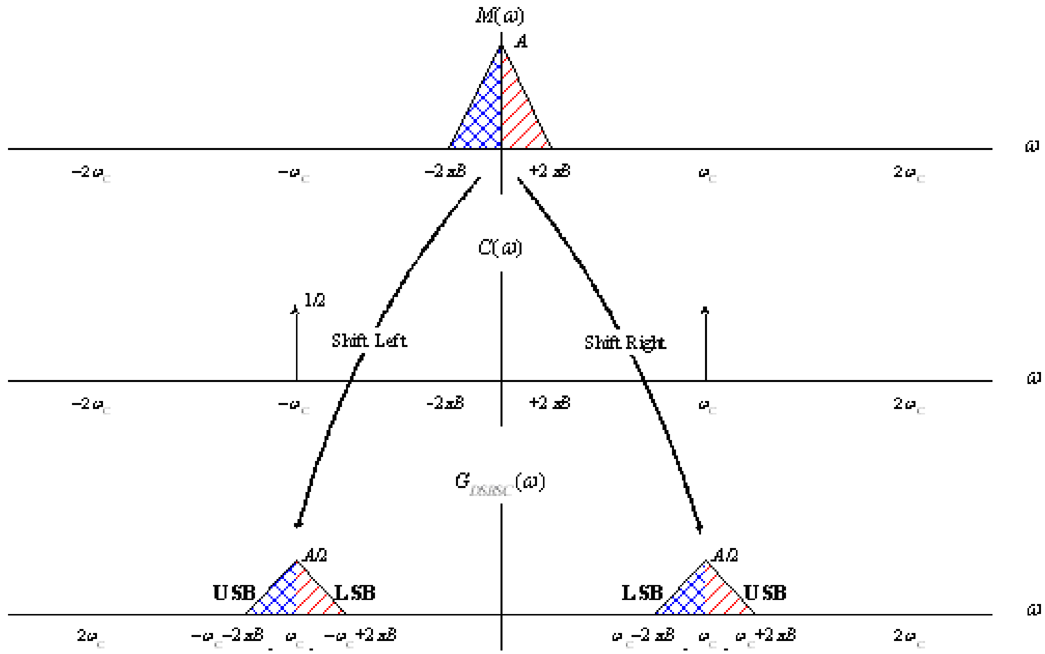
DSBSC carrier is filtered or suppressed or receiver. That's why it is called DSBSC

Problem with DSBSC

1) Geometrical Carrier or Receiver
2) Phase Detection Problem
3) Frequency Shifting Properties

Time and Frequency Representation of DSBSC Modulation Process





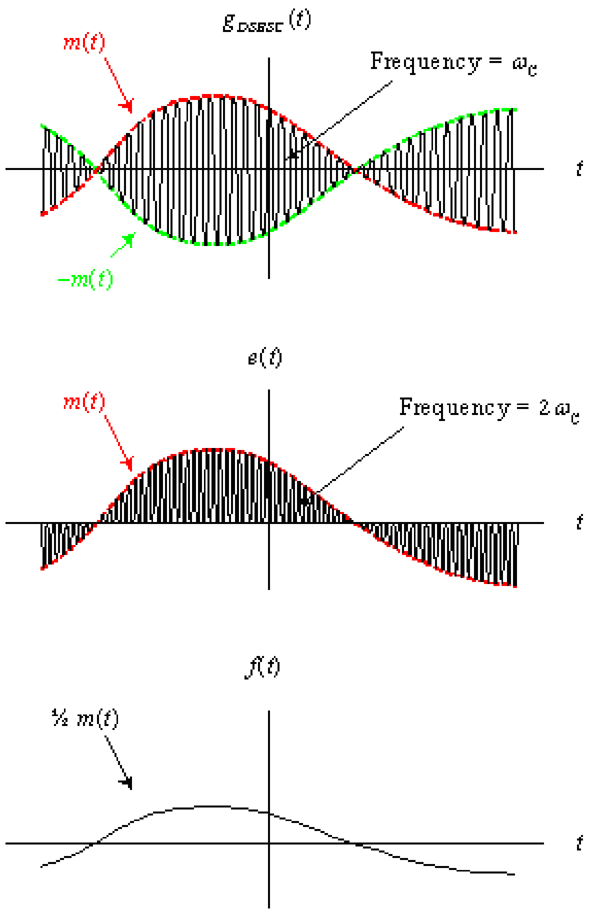
DSBSC Demodulation

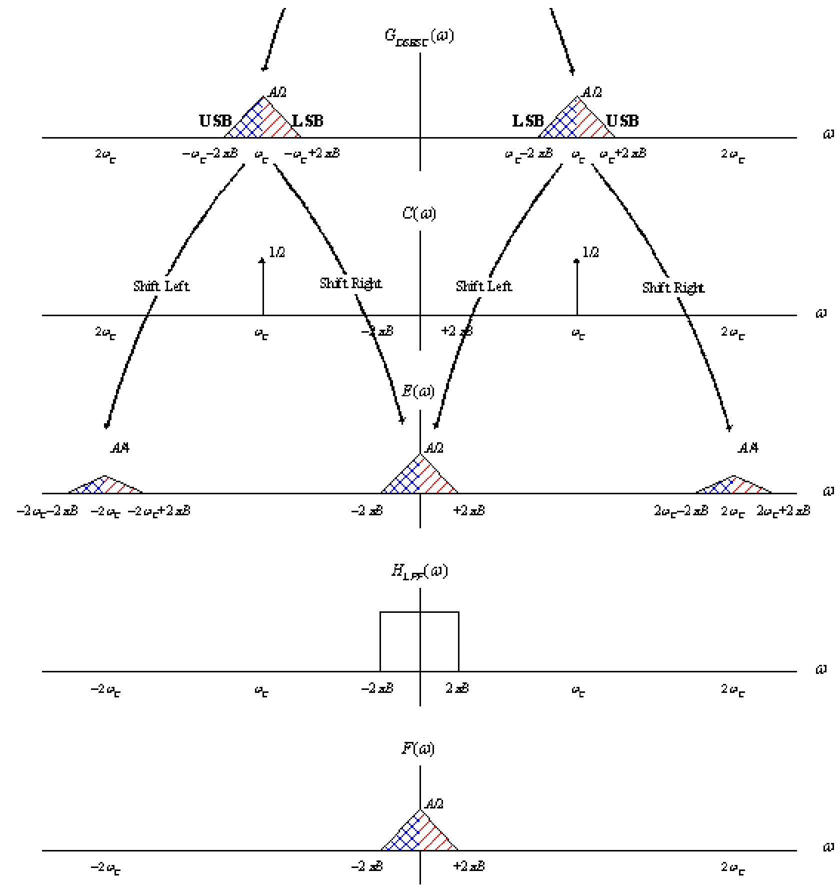
Double Sideband Suppressed Carrier (DSBSC)

For a broadcast system it is more economical to have one experience high power transmitter and expensive receiver, for such application a large carrier signal is transmitted along with the suppressed carrier modulated signal m(t) Cos (wct), thus no need to generate a local carrier. This is called AM in which the transmitted signal is.

$$\text{Cos } wct \; [\; A + m(t) \;]$$

Time and Frequency Representation of DSBSC Demodulation Process





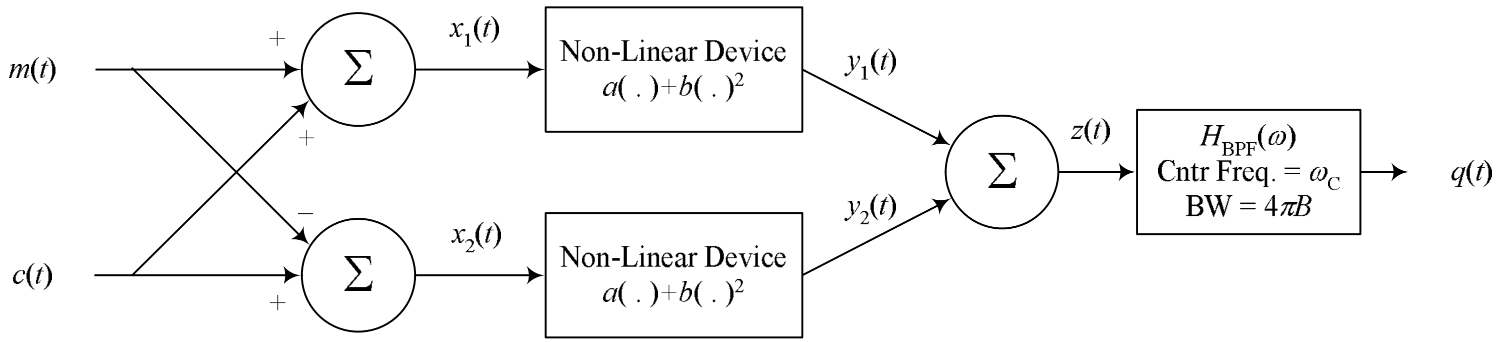
Modulator Circuits

- Basically we are after multiplying a signal with a carrier.
- There are three realizations of this operation:
  - Multiplier Circuits
  - Non-Linear Circuits
  - Switching Circuits

Non-Linear Devices (NLD)

- A NLD is a device whose input-output relation is non-linear. One such example is the diode ($i_D = e^{v_D/v_T}$).
- The output of a NLD can be expressed as a power series of the input, that is
  $y(t) = ax(t) + bx^2(t) + cx^3(t) + \ldots$
- When $x(t) << 1$, the higher powers can be neglected, and the output can be approximated by the first two terms.
- When the input $x(t)$ is the sum of two signal, $m(t)+c(t)$, $x^2(t)$ will have the product term $m(t)c(t)$

Non-Linear Modulators



DSBSC modulation using non-linear device

$$x_1(t) = c(t) + m(t) = \cos(\omega_C t) + m(t)$$

$$x_1(t) = c(t) - m(t) = \cos(\omega_C t) - m(t)$$

$$y_1(t) = a\left[\cos(\omega_C t) + m(t)\right] + b\left[\cos(\omega_C t) + m(t)\right]^2$$

$$= a\cos(\omega_C t) + am(t) + bm^2(t) + 2bm(t)\cdot\cos(\omega_C t) + b\cos^2(\omega_C t)$$

$$= \underbrace{am(t)}_{Undesired} + \underbrace{bm^2(t)}_{Undesired} + \underbrace{2bm(t)\cdot\cos(\omega_C t)}_{Desired} + \underbrace{a\cos(\omega_C t)}_{Undesired} + \underbrace{\frac{b}{2}}_{Undesired} + \underbrace{\frac{b}{2}\cos(2\omega_C t)}_{Undesired}$$

$$z(t) = y_1(t) - y_2(t)$$

$$= \underbrace{2am(t)}_{Undesired} + \underbrace{4bm(t)\cdot\cos(\omega_C t)}_{Desired}$$

$$y_2(t) = a\left[\cos(\omega_C t) - m(t)\right] + b\left[\cos(\omega_C t) - m(t)\right]^2$$

$$= a\cos(\omega_C t) - am(t) + bm^2(t) - 2bm(t)\cdot\cos(\omega_C t) + b\cos^2(\omega_C t)$$

$$= \underbrace{-am(t)}_{Undesired} + \underbrace{bm^2(t)}_{Undesired} - \underbrace{2bm(t)\cdot\cos(\omega_C t)}_{Desired} + \underbrace{a\cos(\omega_C t)}_{Undesired} + \underbrace{\frac{b}{2}}_{Undesired} + \underbrace{\frac{b}{2}\cos(2\omega_C t)}_{Undesired}$$

Switching Modulators

- Any periodic function can be expressed as a series of cosines (Fourier Series).
- The information signal, $m(t)$, can therefore be, equivalently, multiplied by any periodic function, and followed by BPF.
- Let this periodic function be a train of pulses.
- Multiplication by a train of pulses can be realized by simple *switching*.

Switching Modulator Illustration

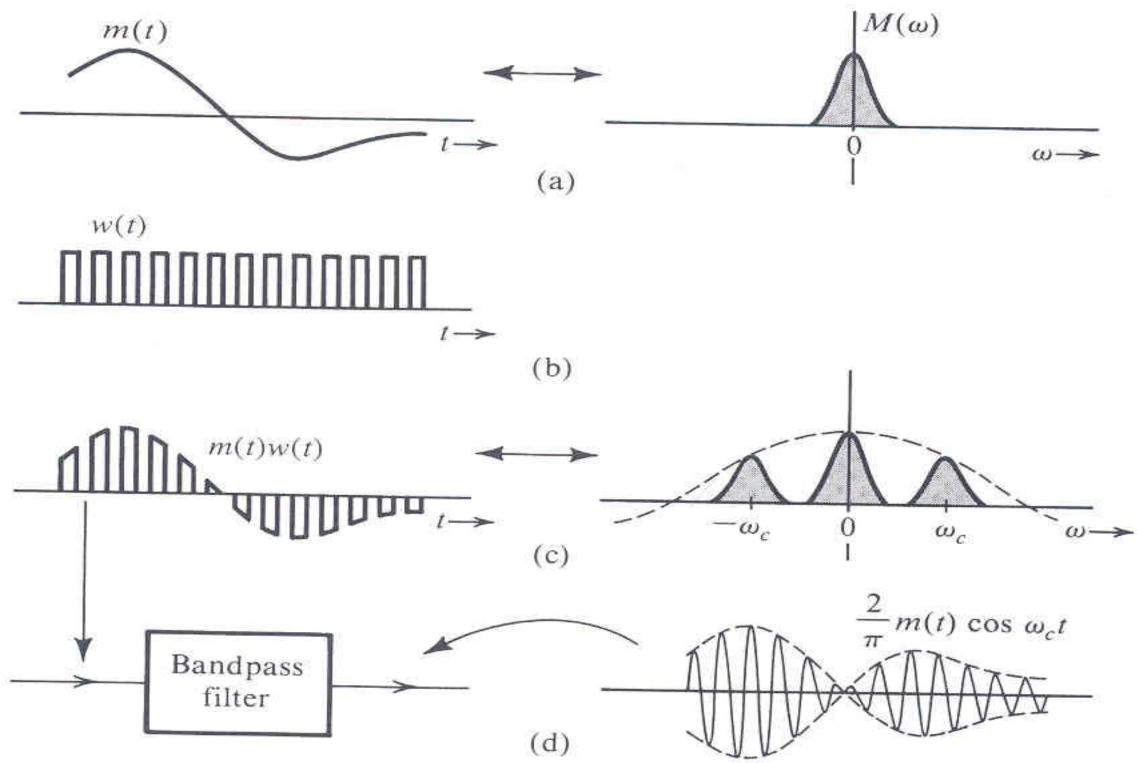


Figure 4.4 Switching modulator for DSB-SC.

Switching Modulator: Diode Bridge

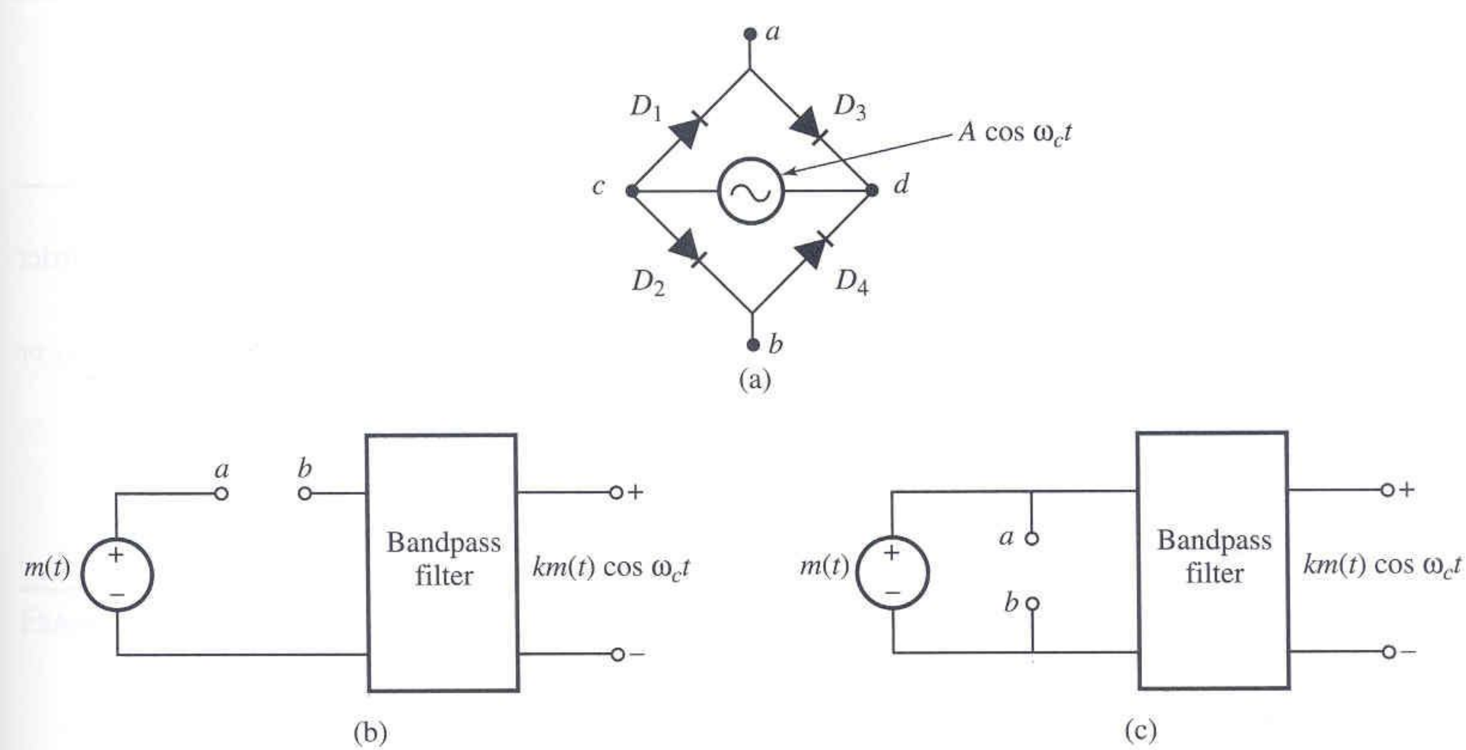


Figure 4.5 (a) Diode-bridge electronic switch. (b) Series-bridge diode modulator. (c) Shunt-bridge diode modulator.

Switching Modulator: Ring

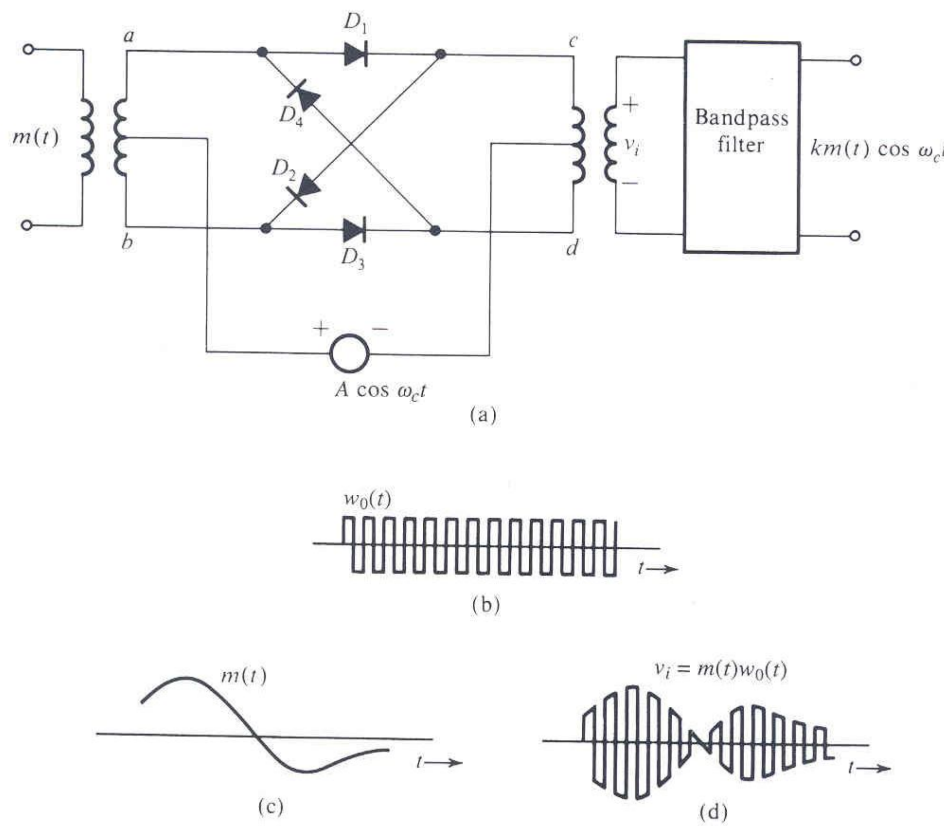
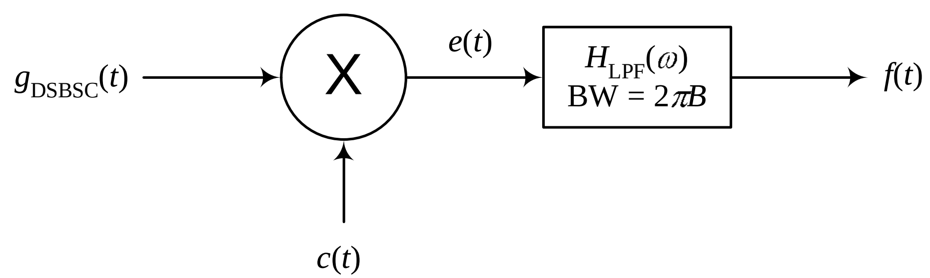


Figure 4.6 Ring modulator.

Demodulation of DSBSC

- The modulator circuits can be used for demodulation, but replacing the BPF by a LPF of bandwidth *B* Hz.
- The receiver must generate a carrier frequency in phase and frequency synchronization with the incoming carrier.
- This type of demodulation is therefore called *coherent* demodulation (or detection).



DSBSC Demodulator (receiver)

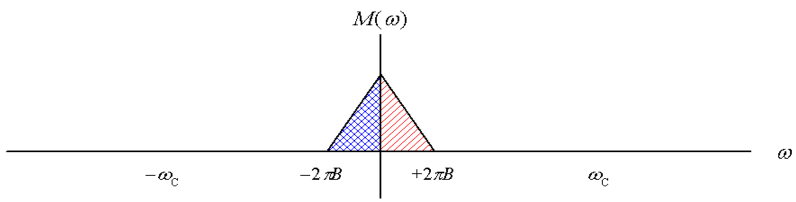
From DSBSC to DSBWC (AM)

- Carrier recovery circuits, which are required for the operation of coherent demodulation, are sophisticated and could be quite costly.
- If we can let $m(t)$ be the envelope of the modulated signal, then a much simpler circuit, the *envelope detector*, can be used for demodulation (non-coherent demodulation).
- How can we make $m(t)$ be the envelope of the modulated signal?

Single-Side Band (SSB) Modulation

- DSBSC (as well as AM) occupies double the bandwidth of the baseband signal, although the two sides carry the same information.
- Why not send only one side, the upper or the lower?
- **Modulation:** similar to DSBSC. Only change the settings of the BPF (center frequency, bandwidth).
- **Demodulation:** similar to DSBSC (coherent)

SSB Representation



How would we represent the SSB signal in the time domain?

$g_{USB}(t) = ?$

$g_{LSB}(t) = ?$

Time-Domain Representation of SSB (1/2)

$M(\omega) = M_+(\omega) + M_-(\omega)$

Let $m_+(t) \leftrightarrow M_+(\omega)$ and $m_-(t) \leftrightarrow M_-(\omega)$

Then: $m(t) = m_+(t) + m_-(t)$ [linearity]

Because $M_+(\omega)$, $M_-(\omega)$ are not even

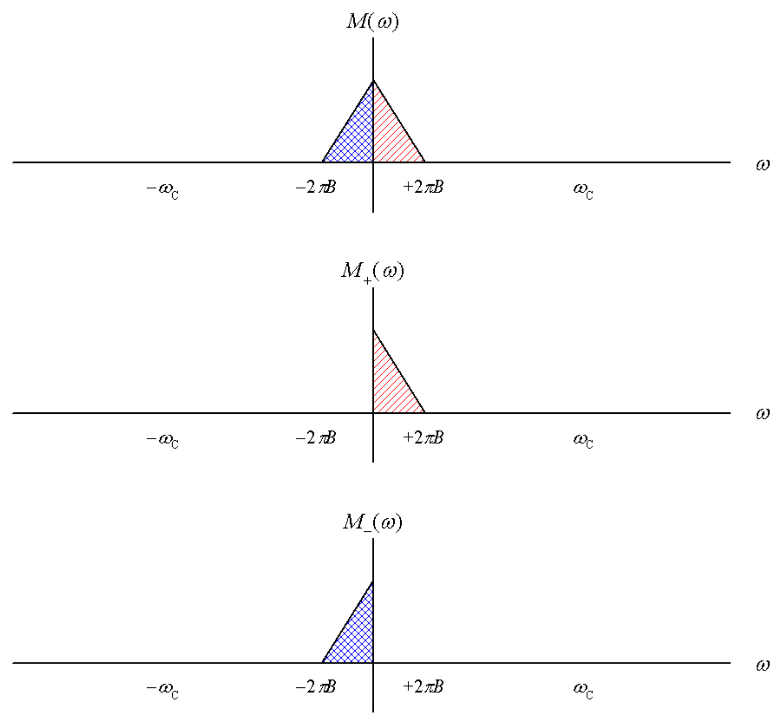
➔ $m_+(t)$, $m_-(t)$ are complex.

Since their sum is real they must be conjugates.

$m_+(t) = ½\, [m(t) + j\, m_h(t)]$

$m_-(t) = ½\, [m(t) - j\, m_h(t)]$

What is $m_h(t)$?



Time-Domain Representation of SSB (2/2)

$M(\omega) = M_+(\omega) + M_-(\omega)$

$M_+(\omega) = M(\omega)u(\omega)$; $M_-(\omega) = M(\omega)u(-\omega)$

$\text{sgn}(\omega)=2u(\omega) -1$ ➔ $u(\omega)= ½ + ½\ \text{sgn}(\omega)$; $u(-\omega) = ½ -½\ \text{sgn}(\omega)$

$M_+(\omega) = ½[M(\omega) + M(\omega)\text{sgn}(\omega)]$

$M_-(\omega) = ½ [M(\omega) - M(\omega)\text{sgn}(\omega)]$

Comparing to:

$m_+(t) = ½ [m(t) + j\, m_h(t)] \leftrightarrow ½ [M(\omega) + j\, M_h(\omega)]$

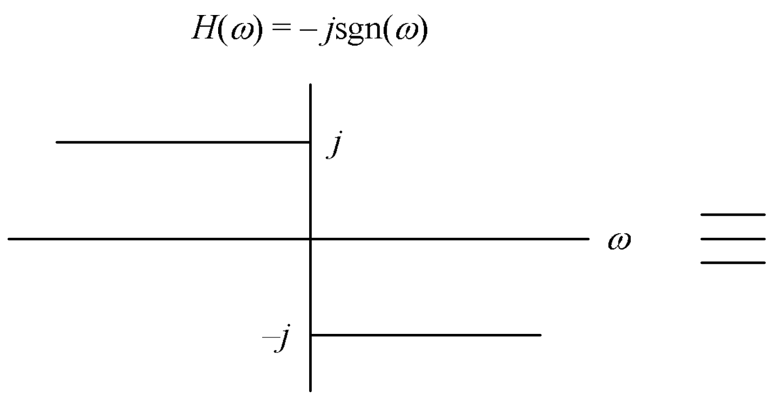
$m_-(t) = ½ [m(t) - j\, m_h(t)] \leftrightarrow ½ [M(\omega) - j\, M_h(\omega)]$

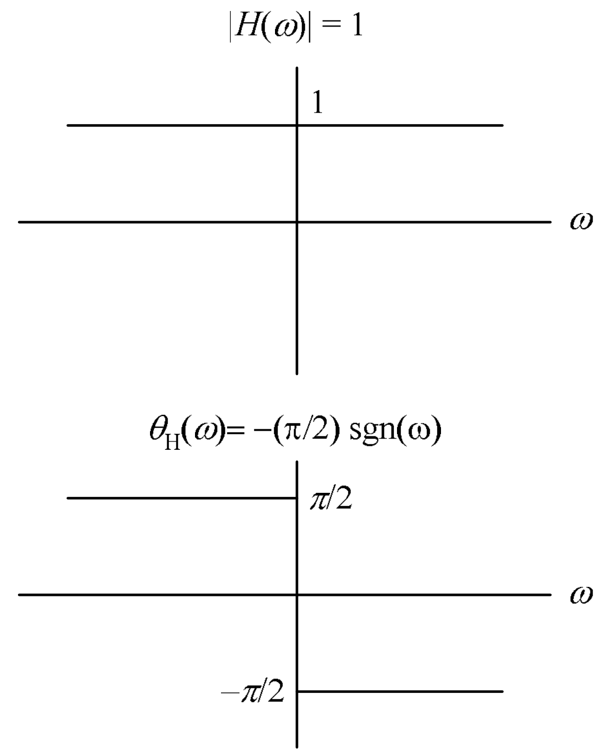
We find

$M_h(\omega) = - j\, M(\omega)\cdot\text{sgn}(\omega)$ where $m_h(t)\leftrightarrow M_h(\omega)$

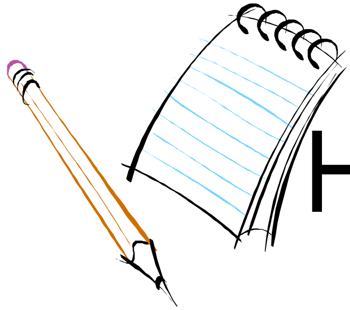
Hilbert Transform

- $m_h(t)$ is known as the Hilbert Transform (HT) of $m(t)$.
- The transfer function of this transform is given by:
  $H(\omega) = -j\,\text{sgn}(\omega)$





- It is basically a $\pi/2$ phase shifter



Hilbert Transform of $\cos(\omega_c t)$

$\cos(\omega_c t) \leftrightarrow \pi\,[\delta(\omega - \omega c) + \delta(\omega + \omega c)]$

$$
\begin{aligned}
HT[\cos(\omega_c t)] &\leftrightarrow -j\,\mathrm{sgn}(\omega)\,\pi\,[\delta(\omega - \omega c) + \delta(\omega + \omega c)] \\
&= \;j\,\mathrm{sgn}(\omega)\,\pi\,[-\delta(\omega - \omega c) - \delta(\omega + \omega c)] \\
&= j\,\pi\,[-\delta(\omega - \omega c) + \delta(\omega + \omega c)] \\
&= j\,\pi\,[\delta(\omega + \omega c) - \delta(\omega - \omega c)] \leftrightarrow \sin(\omega_c t)
\end{aligned}
$$

Which is expected since:

$\cos(\omega_c t - \pi/2) = \sin(\omega_c t)$

Time-Domain Operation for Hilbert Transformation

For Hilbert Transformation $H(\omega) = -j\, \text{sgn}(\omega)$.

What is $h(t)$?

$\text{sgn}(t) \leftrightarrow 2/(j\omega)$ [From FT table]

$2/(jt) \leftrightarrow 2\pi\, \text{sgn}(-\omega)$ [symmetry]

$1/(\pi t) \leftrightarrow -j\, \text{sgn}(\omega)$

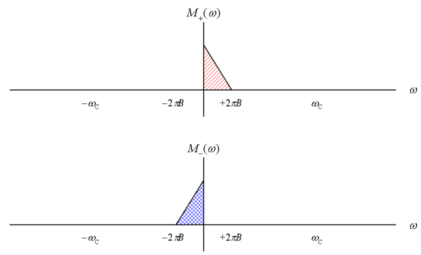
Since $M_h(\omega) = -\ j\, M(\omega) \cdot \text{sgn}(\omega) = H(\omega) \cdot M(\omega)$

Then

$$m_h(t) = \frac{1}{\pi t} * m(t)$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{m(\alpha)}{t-\alpha} d\alpha$$

Finally …



$$g_{USB}(t) = m_+(t)e^{j\omega_C t} + m_-(t)e^{-j\omega_C t}$$

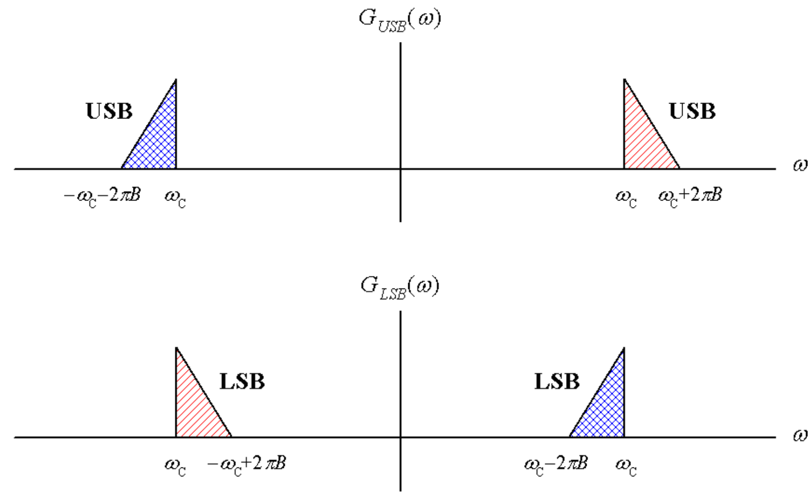
$$g_{LSB}(t) = m_+(t)e^{-j\omega_C t} + m_-(t)e^{j\omega_C t}$$

$$g_{USB}(t) = \frac{1}{2}m(t)e^{j\omega_C t} + \frac{1}{2}jm_h(t)e^{j\omega_C t} + \frac{1}{2}m(t)e^{-j\omega_C t} - \frac{1}{2}jm_h(t)e^{-j\omega_C t}$$

$$= m(t)\cos(\omega_C t) - m_h(t)\sin(\omega_C t)$$

$$g_{LSB}(t) = \frac{1}{2}m(t)e^{j\omega_C t} - \frac{1}{2}jm_h(t)e^{j\omega_C t} + \frac{1}{2}m(t)e^{-j\omega_C t} + \frac{1}{2}jm_h(t)e^{-j\omega_C t}$$

$$= m(t)\cos(\omega_C t) + m_h(t)\sin(\omega_C t)$$



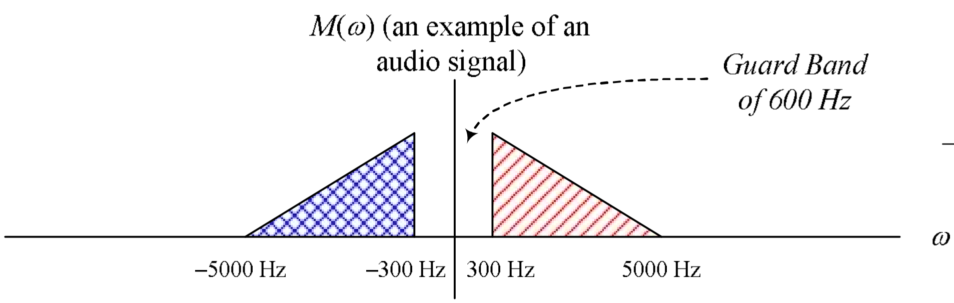
$$G_{USB}(\omega) = M_+(\omega - \omega_C) + M_-(\omega + \omega_C)$$

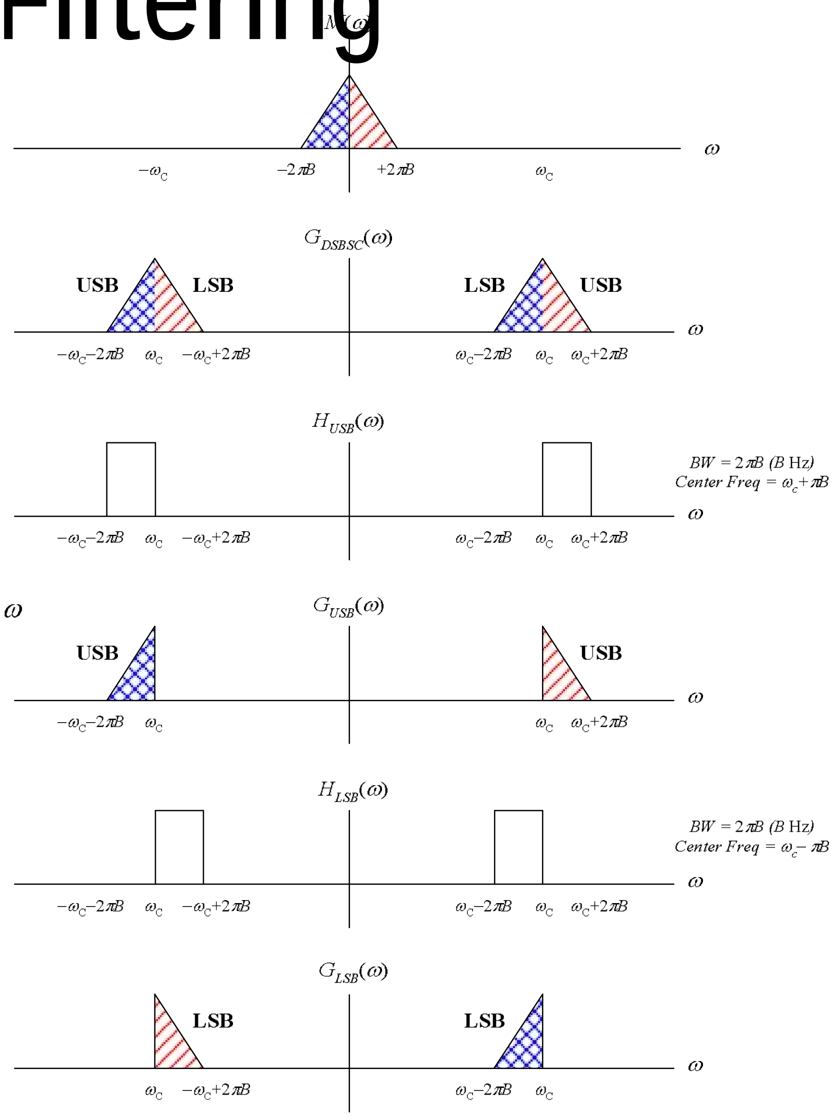
$$G_{LSB}(\omega) = M_+(\omega + \omega_C) + M_-(\omega - \omega_C)$$

Generation of SSB

- Selective Filtering Method
  Realization based on spectrum analysis
- Phase-Shift Method
  Realization based on time-domain expression of the modulated signal

Selective Filtering

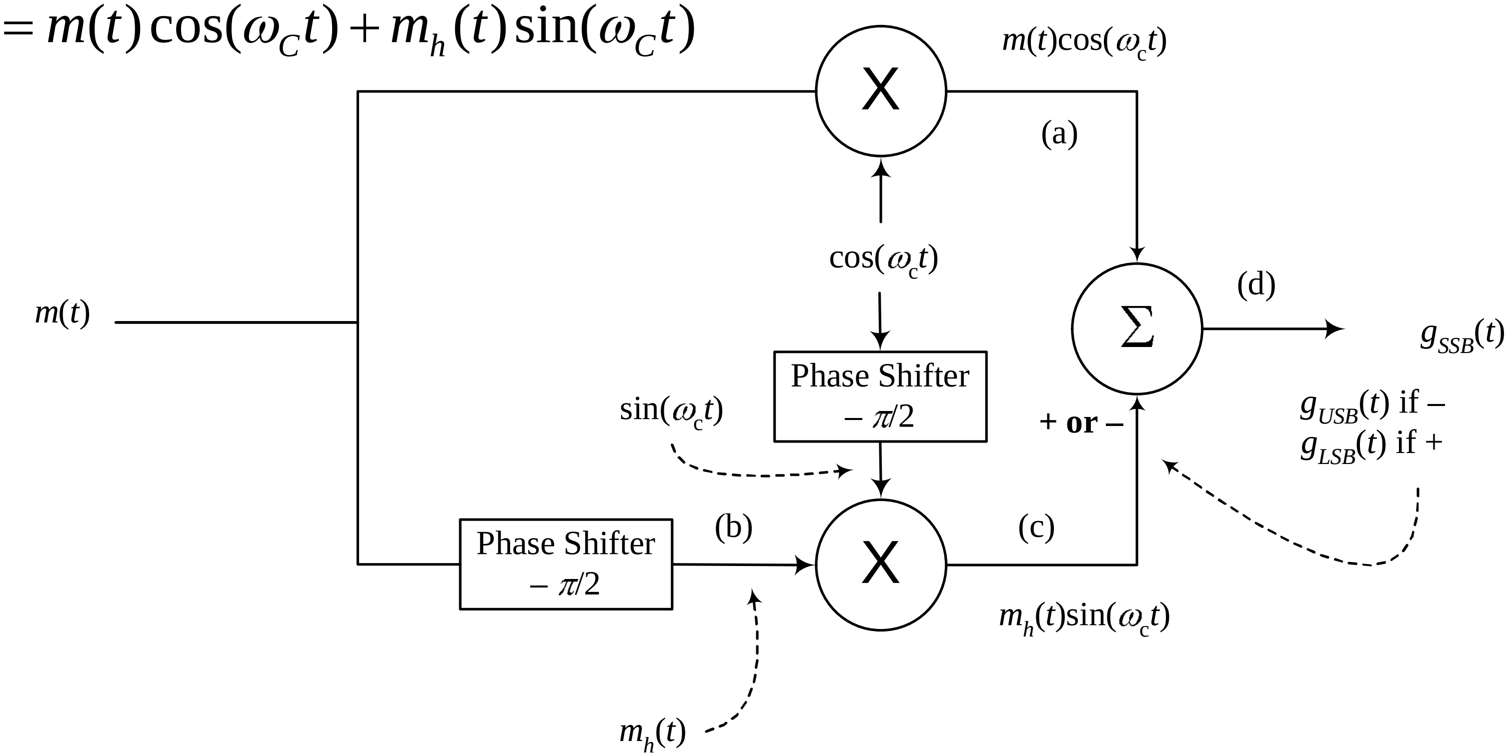




Phase Shifting

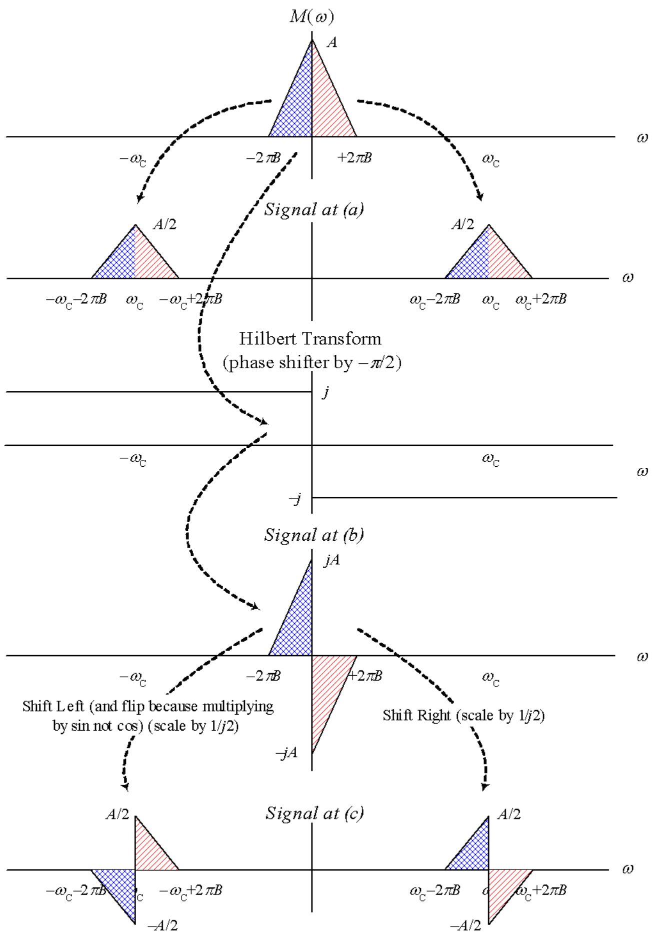
$$g_{USB}(t) = m(t)\cos(\omega_C t) - m_h(t)\sin(\omega_C t)$$

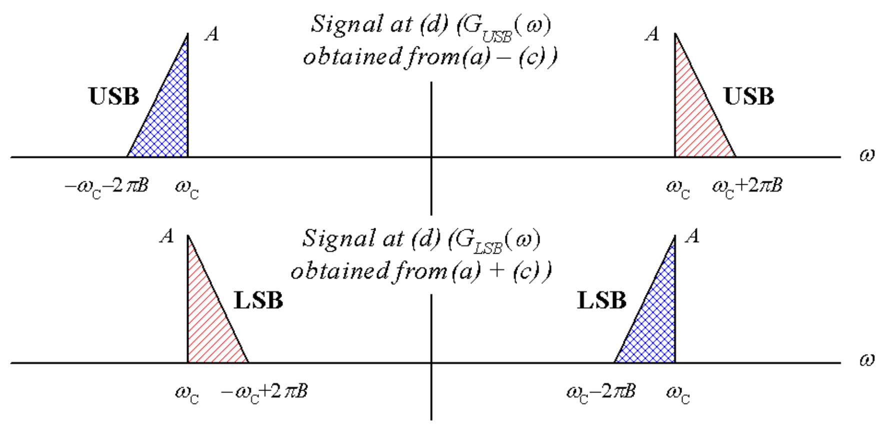
$$g_{LSB}(t) = m(t)\cos(\omega_C t) + m_h(t)\sin(\omega_C t)$$



SSB Modulator

Phase-shifting Method: Frequency-Domain Illustration



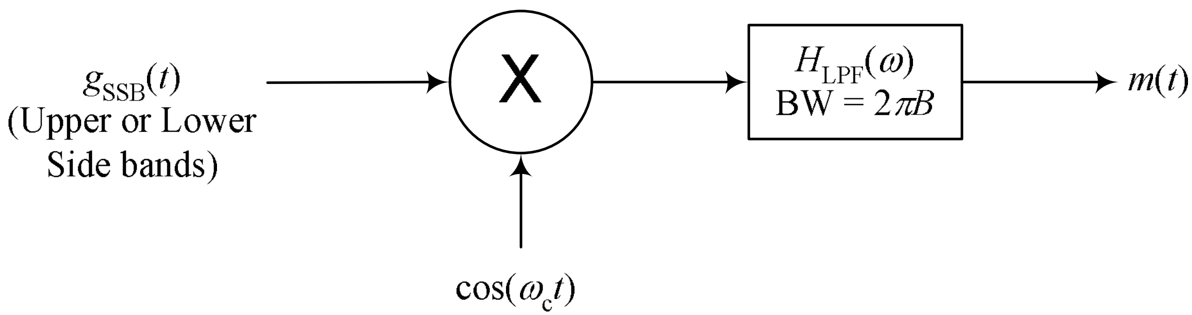


SSB Demodulation (Coherent)

$$g_{SSB}(t) = m(t)\cos(\omega_C t) \pm m_h(t)\sin(\omega_C t)$$

$$g_{SSB}(t)\cos(\omega_C t) = \frac{1}{2}m(t)[1+\cos(2\omega_C t)] \pm \frac{1}{2}m_h(t)\sin(2\omega_C t)$$

$$\text{LPF Output} = \frac{1}{2}m(t)$$

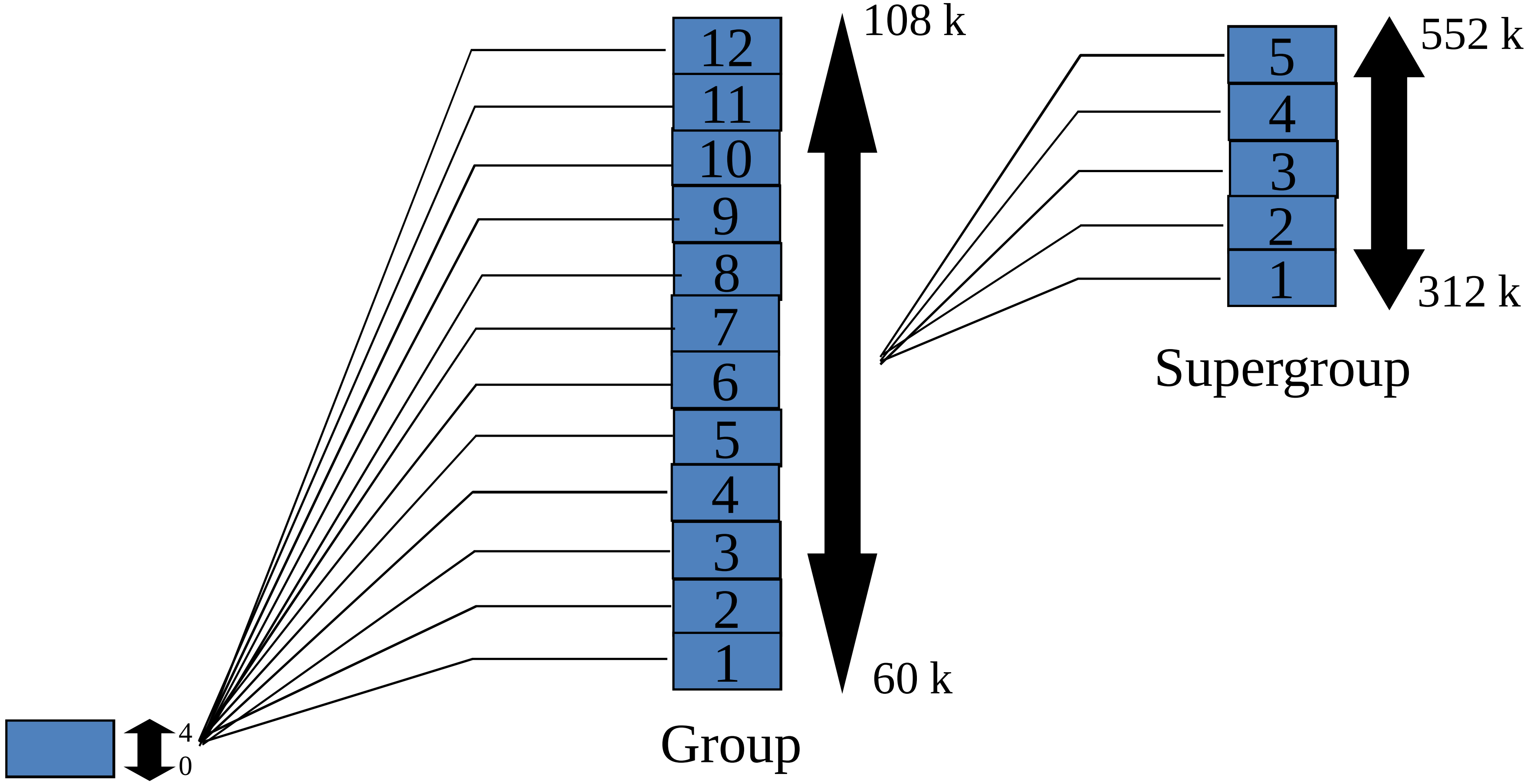


SSB Demodulator (receiver)

FDM in Telephony

- FDM is done in stages
  - Reduce number of carrier frequencies
  - More practical realization of filters
- Group: 12 voice channels × 4 kHz = 48 kHz
  occupy the band 60-108 kHz
- Supergroup: 5 groups × 48 kHz = 240 kHz
  occupy the band 312-552
- Mastergroup: 10 S-G × 240 kHz = 2400 kHz
  occupy the band 564-3084 kHz

FDM Hierarchy



Vestigial Side Band Modulation (VSB)

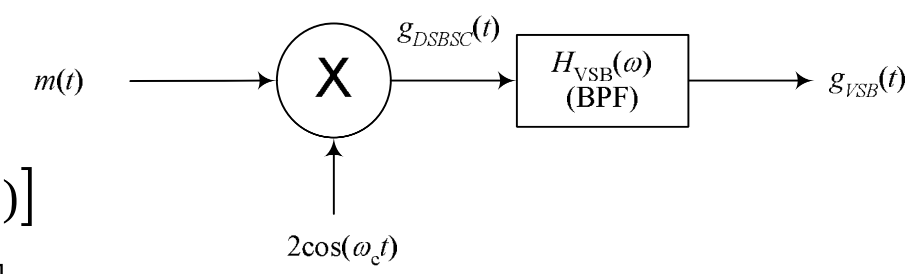
- What if we want to generate SSB using selective filtering but there is no guard band between the two sides?
  ➔We will filter-in a *vestige* of the other band.
- Can we still recover our message, without distortion, after demodulation?
  Yes. If we use a proper LPF.

Filtering Condition of VSB

$$g_{DSBSC}(t) = 2m(t)\cos(\omega_C t)$$

$$G_{DSBSC}(\omega) = M(\omega - \omega_C) + M(\omega + \omega_C)$$

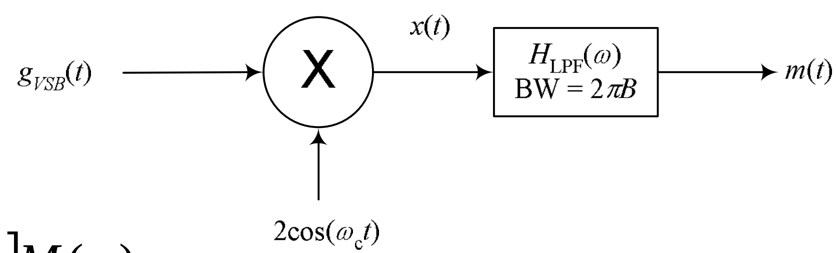
$$G_{VSB}(\omega) = H_{VSB}(\omega)\left[M(\omega - \omega_C) + M(\omega + \omega_C)\right]$$



VSB Modulator (transmitter)

$$X(\omega) = H_{VSB}(\omega - \omega_C)\left[\underbrace{M(\omega - 2\omega_C)}_{at\ +2\omega_C} + \underbrace{M(\omega)}_{Baseband}\right]$$

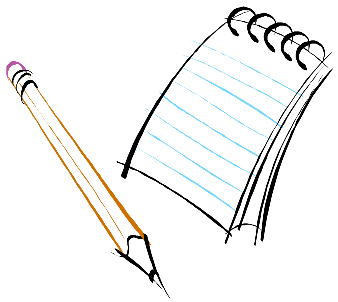
$$+ H_{VSB}(\omega + \omega_C)\left[\underbrace{M(\omega)}_{baseband} + \underbrace{M(\omega + 2\omega_C)}_{at\ -2\omega_C}\right]$$



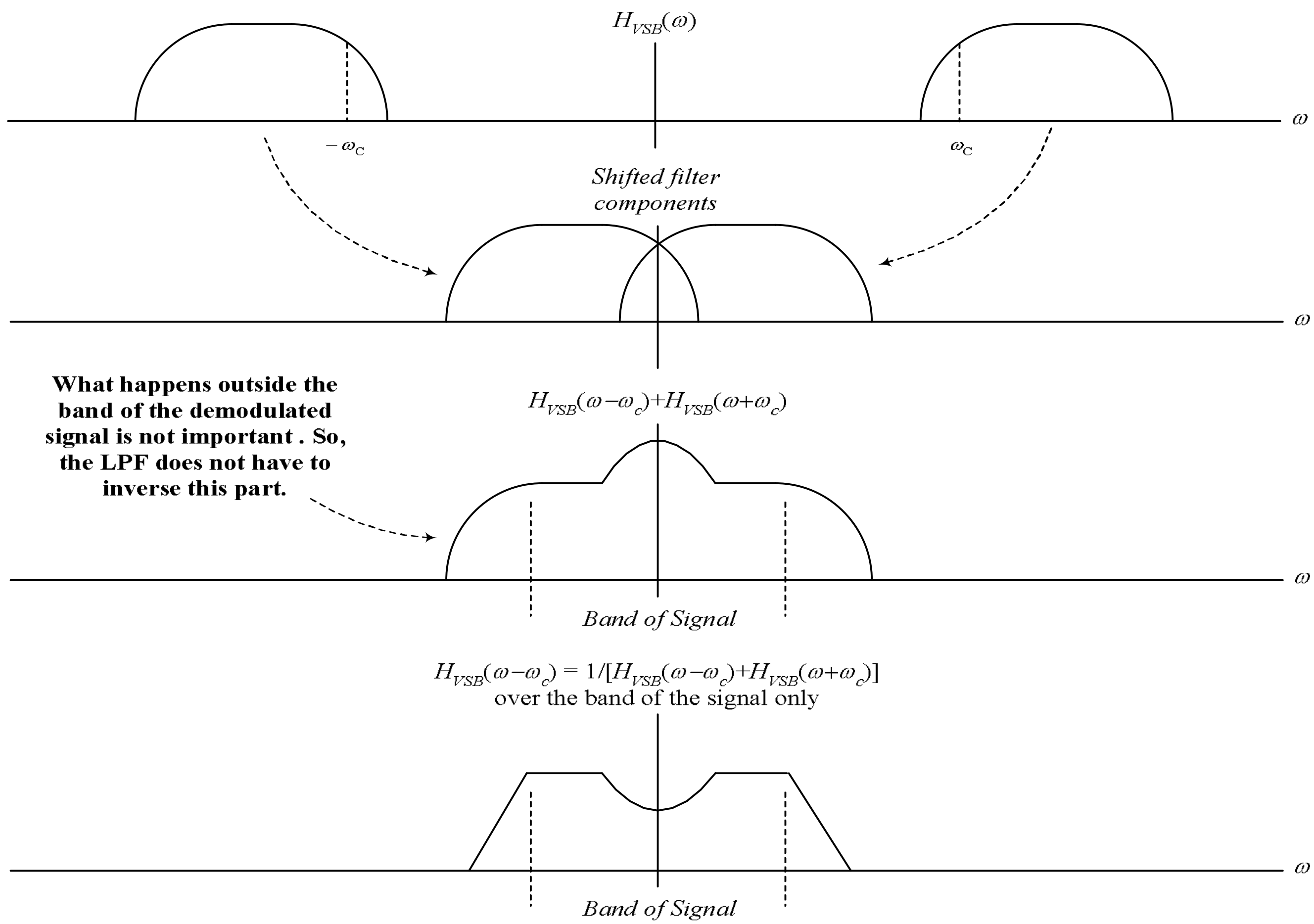
VSB Demodulator (receiver)

$$Z(\omega) = H_{LPF}(\omega)\left[H_{VSB}(\omega - \omega_C) + H_{VSB}(\omega + \omega_C)\right]M(\omega)$$

$$H_{LPF}(\omega) = \frac{1}{H_{VSB}(\omega - \omega_C) + H_{VSB}(\omega + \omega_C)} \quad ; |\omega| \le 2\pi B$$



VSB Filtering



VSB Filter: Special Case

- Condition For distortionless demodulation:

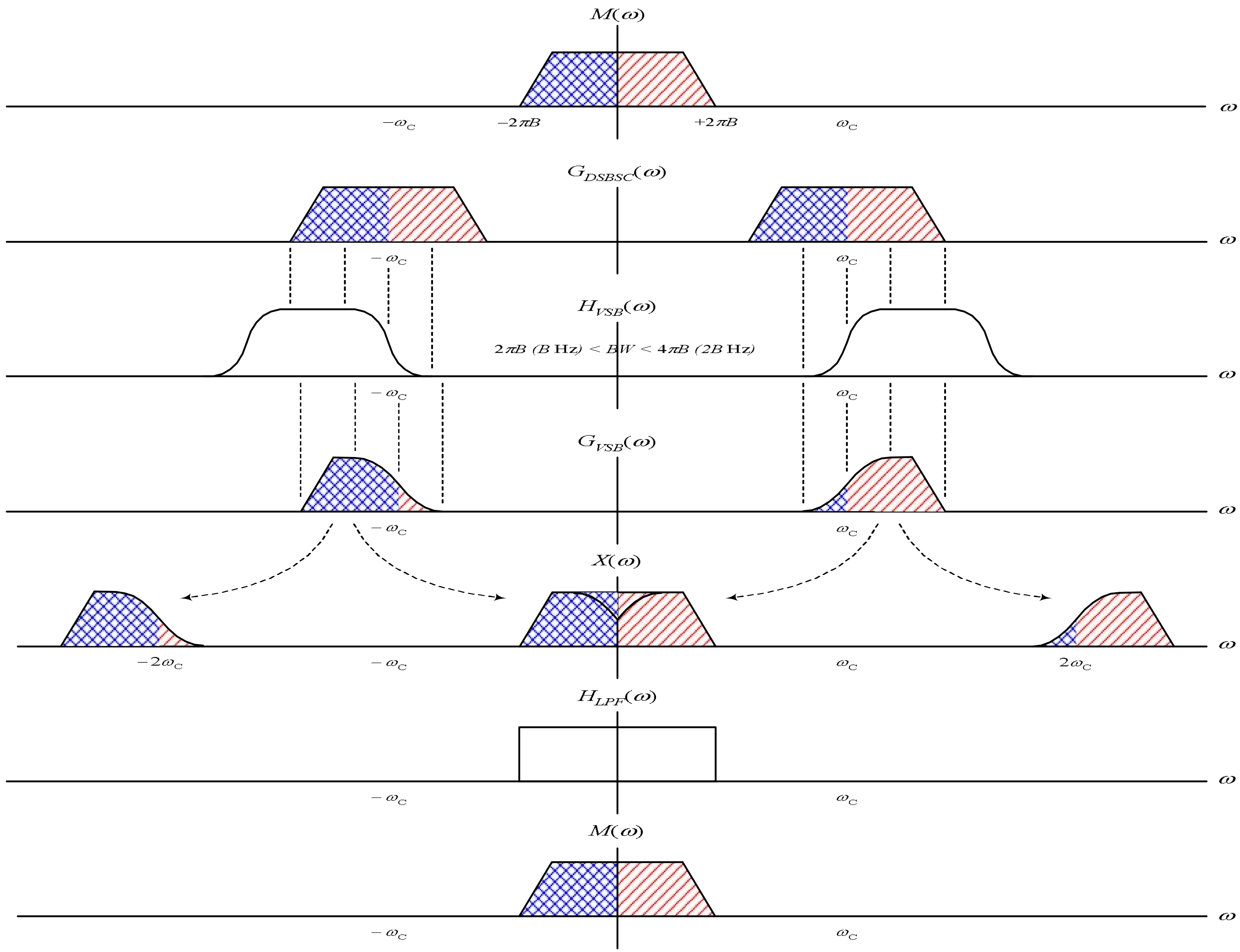
$$H_{LPF}(\omega) = \frac{1}{H_{VSB}(\omega - \omega_C) + H_{VSB}(\omega + \omega_C)} \quad ; |\omega| \leq 2\pi B$$

- If we impose the condition on the filter at the modulator:

$$H_{VSB}(\omega - \omega_c) + H_{VSB}(\omega + \omega_c) = 1 \; ; \; |\omega| \leq 2\pi B$$

Then $H_{LPF} = 1$ for $|\omega| \leq 2\pi B$ (Ideal LPF)

- $H_{VSB}(\omega)$ will then have odd symmetry around ω_c over the transition period.



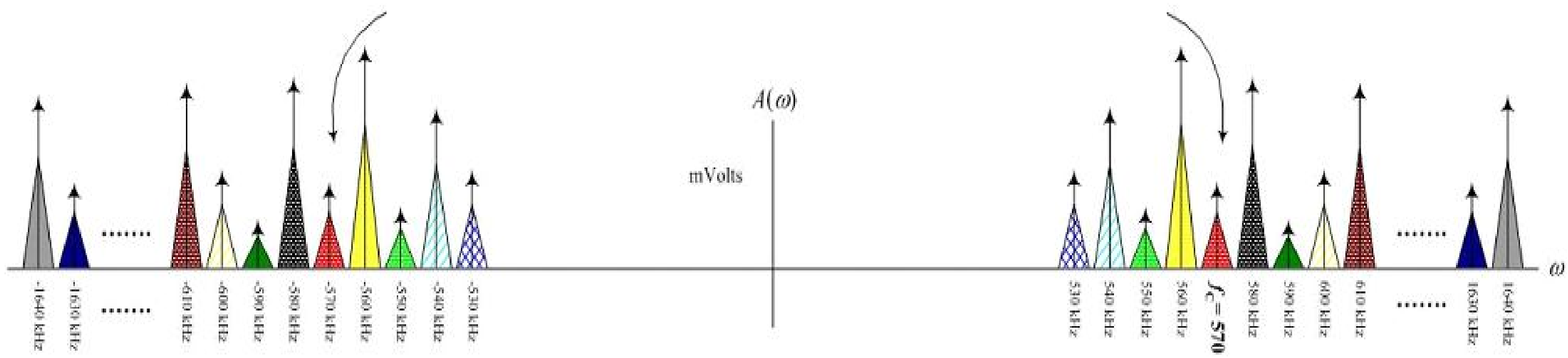
M(ω)
−ωC
−2πB
+2πB
ωC
ω
GDSBSC(ω)
− ωC
ωC
HVSB(ω)
2πB (B Hz) < BW < 4πB (2B Hz)
GVSB(ω)
X(ω)
−2ωC
2ωC
HLPF(ω)

AM Broadcasting

- Allocated the band 530 kHz – 1600 kHz (with minor variations)
- 10 kHz per channel. (9 kHz in some countries)
- More that 100 stations can be licensed in the same geographical area.
- Uses AM modulation (DSB + C)

AM station Reception

- In theory, any station can be extracted from the stream of spectra by tuning the receiver BPF to its center frequency. Then demodulated.



- Impracticalities:
  - Requires a BPF with very high Q-factor (Q = f_c / B).
  - Particularly difficult if the filter is to be tunable.

Angle Modulation – Frequency Modulation

Consider again the general carrier $v_c(t) = V_c \cos(\omega_c t + \varphi_c)$

$(\omega_c t + \varphi_c)$ represents the angle of the carrier.

There are two ways of varying the angle of the carrier.

a) By varying the frequency, ω_c – **Frequency Modulation.**
b) By varying the phase, ϕ_c – **Phase Modulation**

Frequency Modulation

In FM, the message signal $m(t)$ controls the frequency f_c of the carrier. Cons carrier

$$v_c(t) = V_c \cos(\omega_c t)$$

then for FM we may write:

FM signal $v_s(t) = V_c \cos(2\pi(f_c + \text{frequency deviation})t)$,where the frequency deviation will depend on $m(t)$.

Given that the carrier frequency will change we may write for an instantane carrier signal

$$V_c \cos(\omega_i t) = V_c \cos(2\pi f_i t) = V_c \cos(\varphi_i)$$

where ϕ_i is the instantaneous angle $\omega_i t = 2\pi f_i t$ and f_i is the instantaneous frequency.

Frequency Modulation

Since $\varphi_i = 2\pi f_i t$ then $\frac{d\varphi_i}{dt} = 2\pi f_i$ or $f_i = \frac{1}{2\pi}\frac{d\varphi_i}{dt}$

i.e. frequency is proportional to the rate of change of angle.

If f_c is the unmodulated carrier and f_m is the modulating frequency, then we deduce that

$$f_i = f_c + \Delta f_c \cos(\omega_m t) = \frac{1}{2\pi}\frac{d\varphi_i}{dt}$$

Δf_c is the peak deviation of the carrier.

Hence, we have $\frac{1}{2\pi}\frac{d\varphi_i}{dt} = f_c + \Delta f_c \cos(\omega_m t)$,*i.e.* $\frac{d\varphi_i}{dt} = 2\pi f_c + 2\pi \Delta f_c \cos(\omega_m t)$

Frequency Modulation

After integration *i.e.* $\int \left(\omega_c + 2\pi \Delta f_c \cos(\omega_m t)\right) dt$

$$\varphi_i = \omega_c t + \frac{2\pi \Delta f_c \sin(\omega_m t)}{\omega_m}$$

$$\varphi_i = \omega_c t + \frac{\Delta f_c}{f_m} \sin(\omega_m t)$$

Hence for the FM signal, $v_s(t) = V_c \cos(\varphi_i)$

$$v_s(t) = V_c \cos\left(\omega_c t + \frac{\Delta f_c}{f_m} \sin(\omega_m t)\right)$$

Frequency Modulation

The ratio $\frac{\Delta f_c}{f_m}$ is called the **Modulation Index** denoted by β *i.e.*

$$\beta = \frac{\text{Peak frequency deviation}}{\text{modulating frequency}}$$

Note – FM, as implicit in the above equation for $v_s(t)$, is a non-linear process
the principle of superposition does not apply. The FM signal for a message *m*
band of signals is very complex. Hence, $m(t)$ is usually considered as a 'singl
modulating signal' of the form

$$m(t) = V_m \cos(\omega_m t)$$

Frequency Modulation

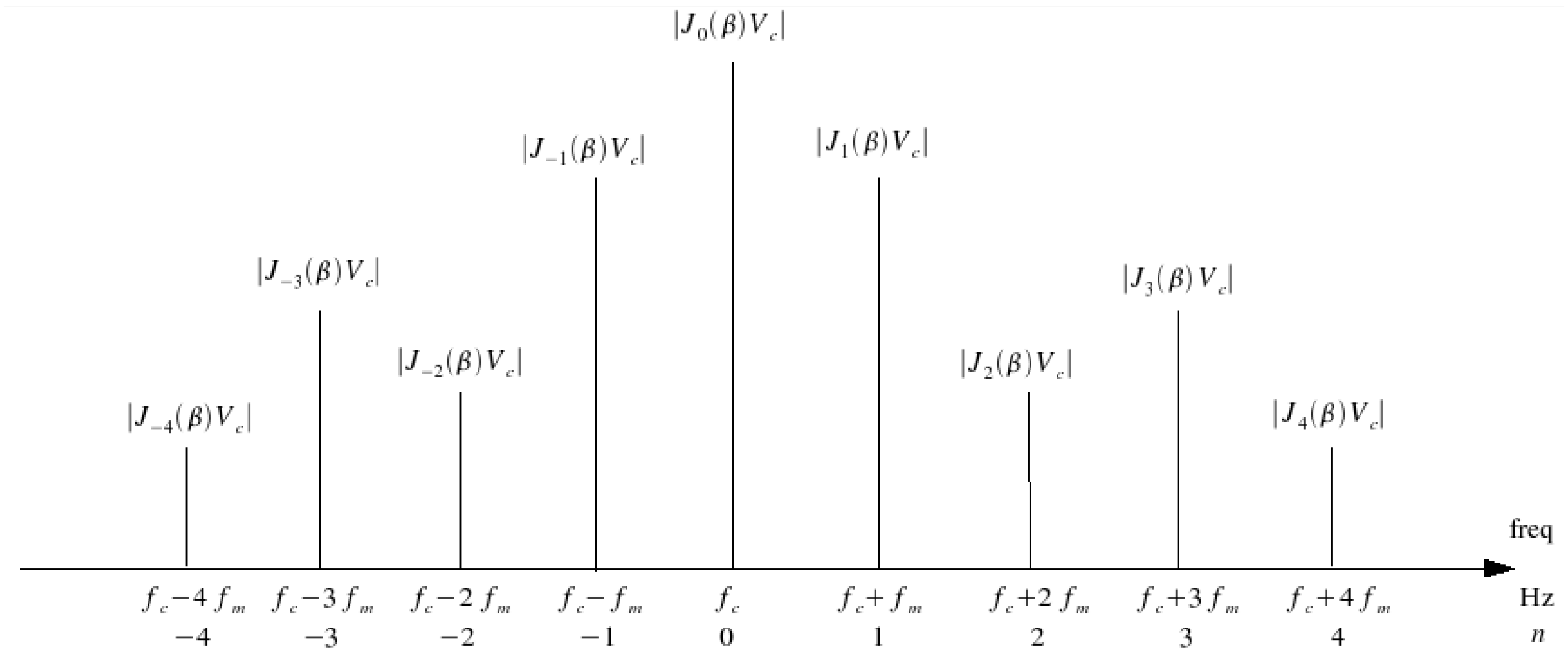
The equation $v_s(t) = V_c \cos\left(\omega_c t + \frac{\Delta f_c}{f_m}\sin(\omega_m t)\right)$ may be expressed as Bessel series (Bessel functions)

$$v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

where $J_n(\beta)$ are Bessel functions of the first kind. Expanding the equation for terms we have:

$$v_s(t) = \underbrace{V_c J_0(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c)}_{f_c}t + \underbrace{V_c J_1(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c + \omega_m)}_{f_c + f_m}t + \underbrace{V_c J_{-1}(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c - \omega_m)}_{f_c - f_m}t$$
$$+ \underbrace{V_c J_2(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c + 2\omega_m)}_{f_c + 2f_m}t + \underbrace{V_c J_{-2}(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c - 2\omega_m)}_{f_c - 2f_m}t + \cdots$$

FM Signal Spectrum.



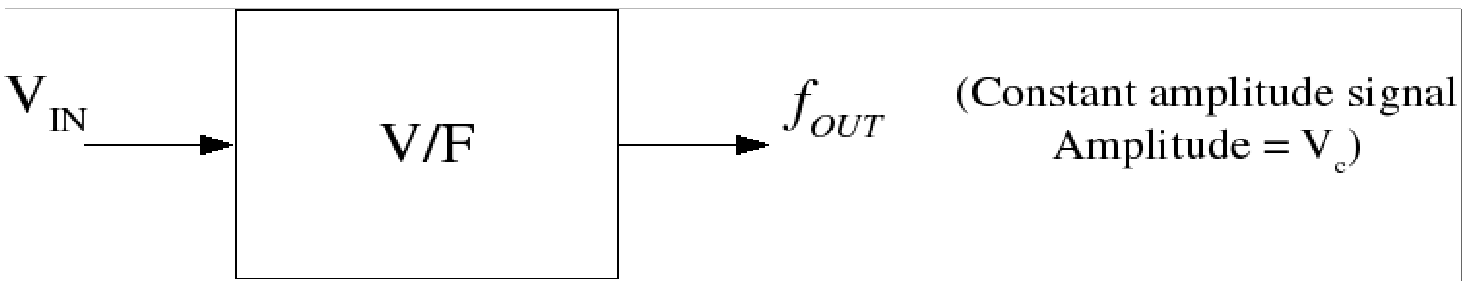
The amplitudes drawn are completely arbitrary, since we have not found any $J_n(\beta)$ – this sketch is only to illustrate the spectrum.

Generation of FM signals – Frequency Modulation.

An FM demodulator is:

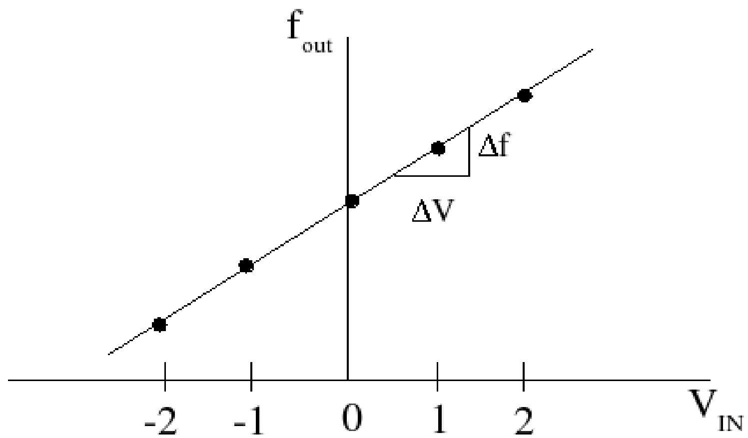
- a voltage-to-frequency converter V/F
- a voltage controlled oscillator VCO

In these devices (V/F or VCO), the output frequency is dependent on the inp amplitude.



V/F Characteristics.

Apply V_{IN} , *e.g.* 0 Volts, +1 Volts, +2 Volts, -1 Volts, -2 Volts, ... and measur frequency output for each V_{IN} . The ideal V/F characteristic is a straight line a shown below.

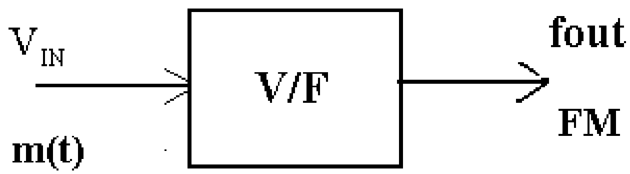


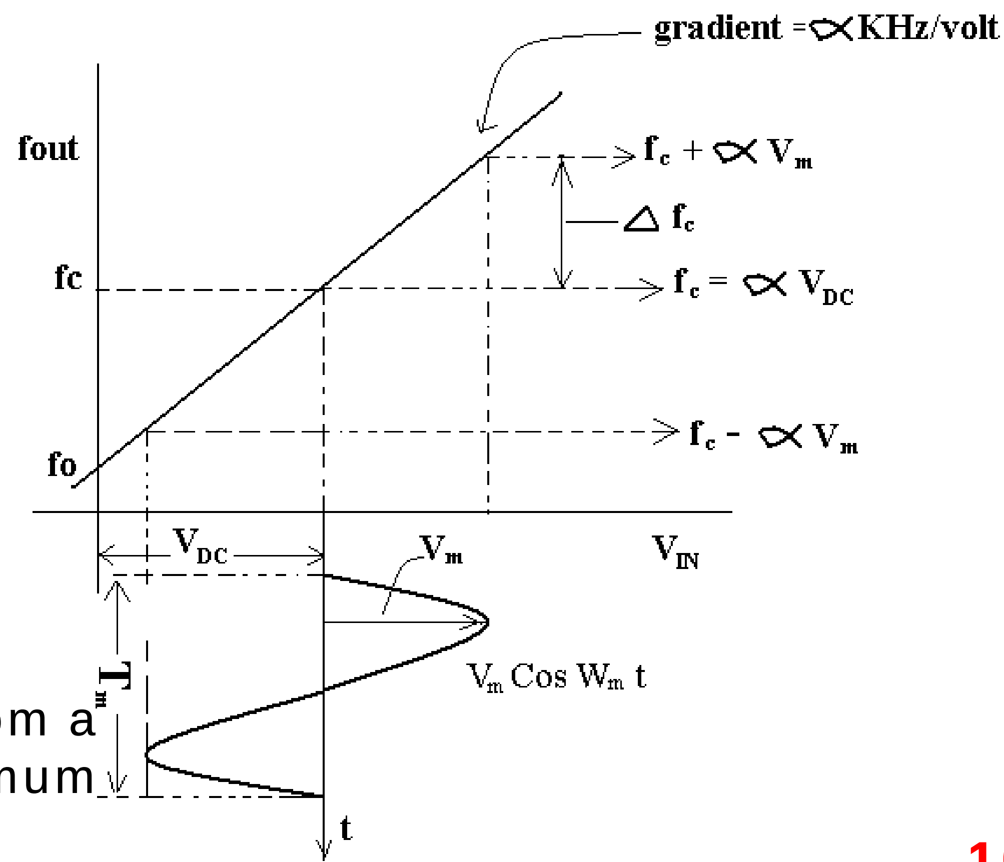
f_c, the frequency output when the input is zero is called the undeviated or no carrier frequency.

The gradient of the characteristic $\frac{\Delta f}{\Delta V}$ is called the **Frequency Conversion Fac** denoted by α per Volt.

V/F Characteristics.

Consider now, an analogue message input, $m(t) = V_m \cos(\omega_m t)$





As the input $m(t)$ varies from

$$+V_m \rightarrow 0 \rightarrow -V_m$$

the output frequency will vary from a maximum, through f_c, to a minimum frequency.

V/F Characteristics.

For a straight line, $y = c + mx$, where c = value of y when $x = 0$, m = gradie
we may say

$$f_{\text{OUT}} = f_c + \alpha V_{\text{IN}}$$

and when $V_{IN} = m(t)$ $f_{\text{OUT}} = f_c + \alpha m(t)$ *,i.e.* the deviation depends on $m(t)$.

Considering that maximum and minimum input amplitudes are $+V_m$ and $-V_m$ respectively, then

$$f_{\max} = f_c + \alpha V_m$$
$$f_{\min} = f_c - \alpha V_m$$

on the diagram on the previous slide.

The peak-to-peak deviation is $f_{max} - f_{min}$, but more importantly for FM the pe
deviation Δf_c is

Peak Deviation, $\Delta f_c = \alpha V_m$ Hence, **Modulation Index**, $\beta = \frac{\Delta f_c}{f_m} = \frac{\alpha V_m}{f_m}$

Summary of the important points of FM

- In FM, the message signal $m(t)$ is assumed to be a single tone frequency,

$$m(t) = V_m \cos(\omega_m t)$$

- The FM signal $v_s(t)$ from which the spectrum may be obtained as

$$v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

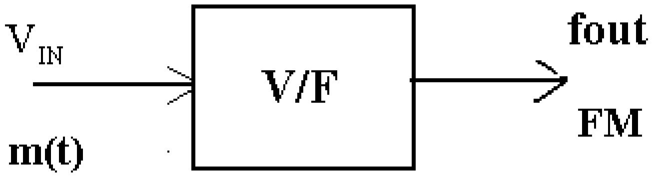
where $J_n(\beta)$ are Bessel coefficients and **Modulation Index**, $\beta = \dfrac{\Delta f_c}{f_m} = \dfrac{\alpha V_m}{f_m}$

- α Hz per Volt is the V/F modulator, gradient or **Frequency Conversion Factor**, α per Volt
- α is a measure of the change in output frequency for a change in input amp

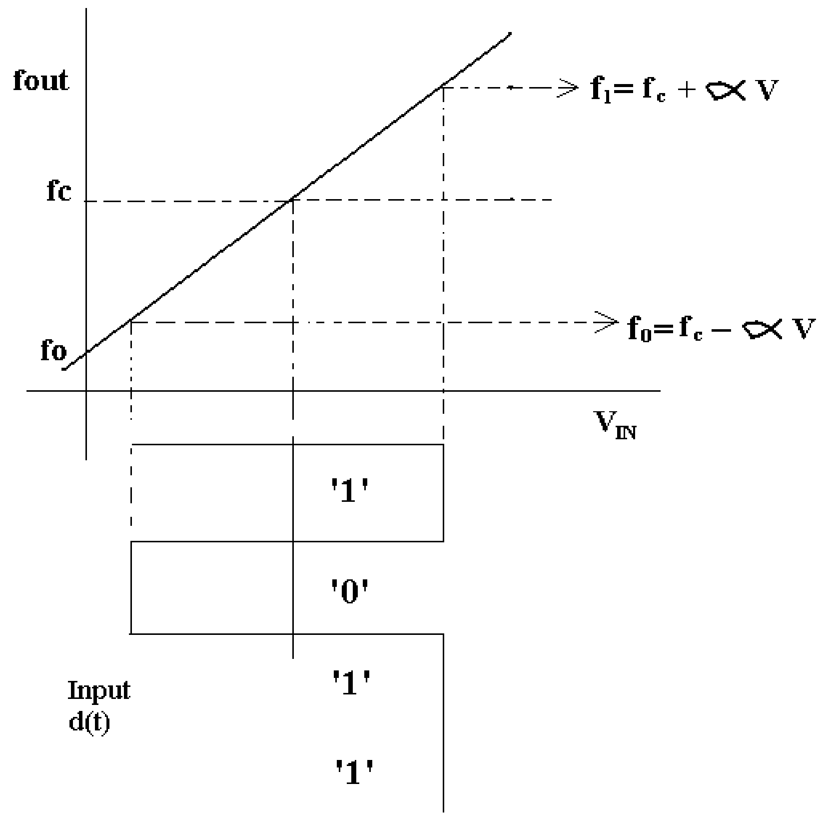
Peak Deviation (of the carrier frequency from f_c) $\Delta f_c = \alpha V_m$

FM Signal Waveforms.

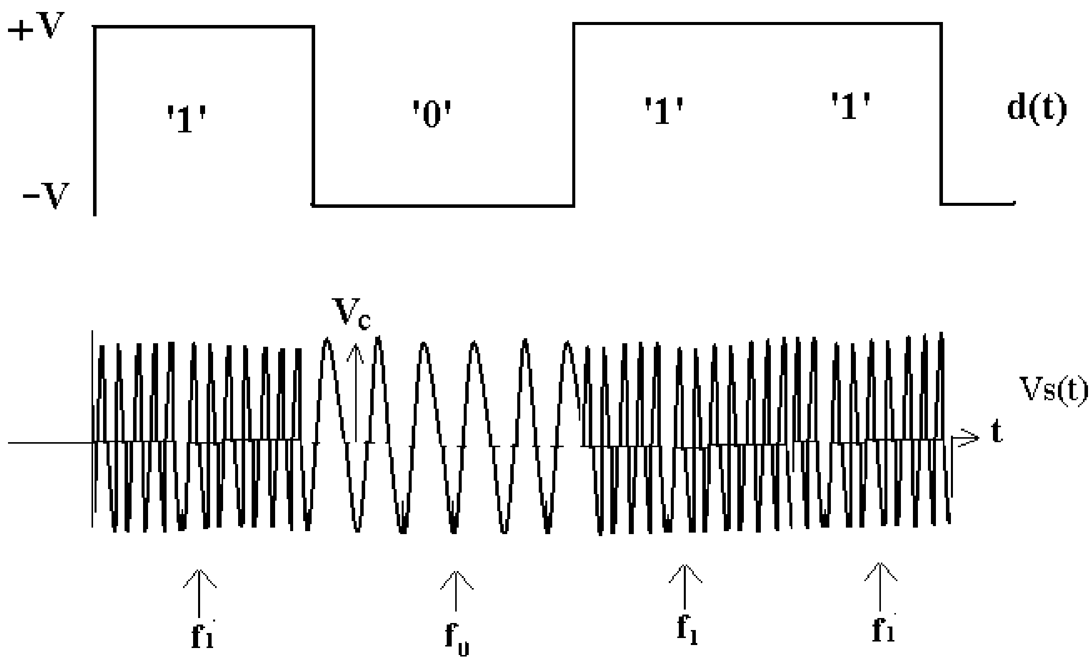
The diagrams below illustrate FM signal waveforms for various inputs



At this stage, an input digital data sequence, *d*(*t*), is introduced – the output in this case will be FSK, (Frequency Shift Keying).



FM Signal Waveforms.

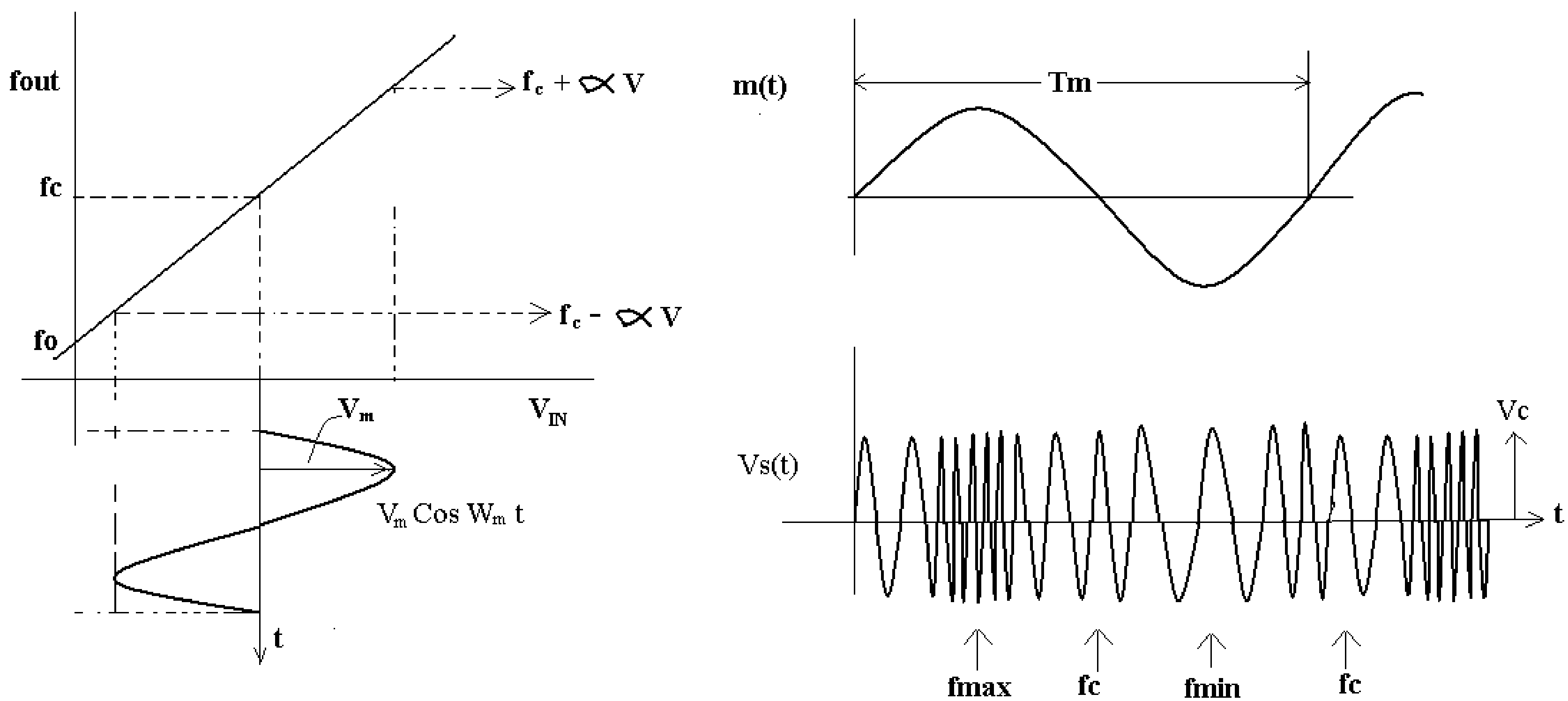


Assuming $d(t) = +V$ for 1's $\quad f_{OUT} = f_1 = f_c + \alpha V$ for 1's

$= -V$ for 0's $\quad f_{OUT} = f_0 = f_c - \alpha V$ for 0's

the output 'switches' between f_1 and f_0.

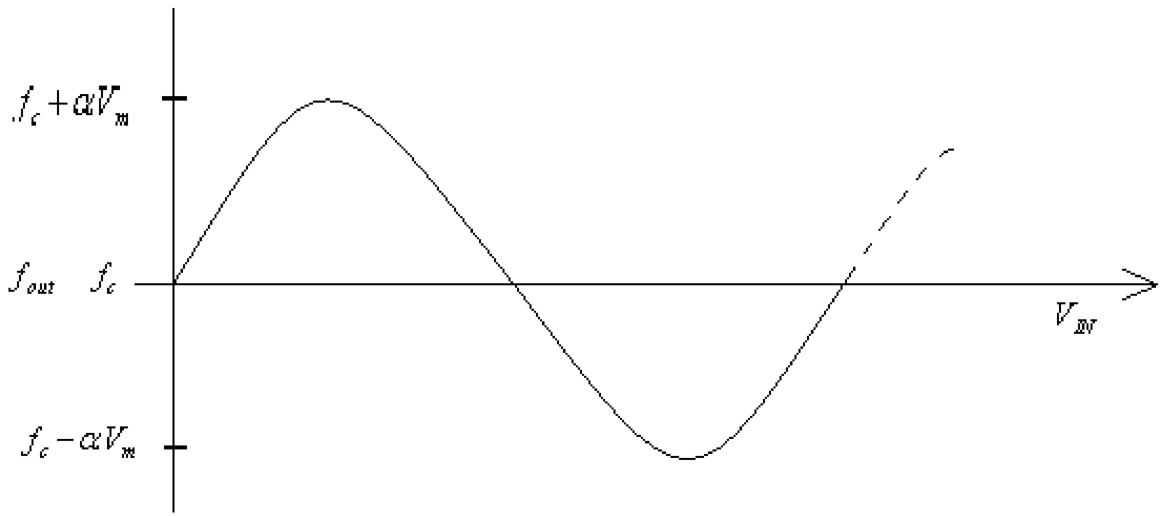
FM Signal Waveforms.



The output frequency varies 'gradually' from f_c to ($f_c + \alpha V_m$), through f_c to ($f_c - \alpha V_m$) *etc.*

FM Signal Waveforms.

If we plot f_{OUT} as a function of V_{IN}:

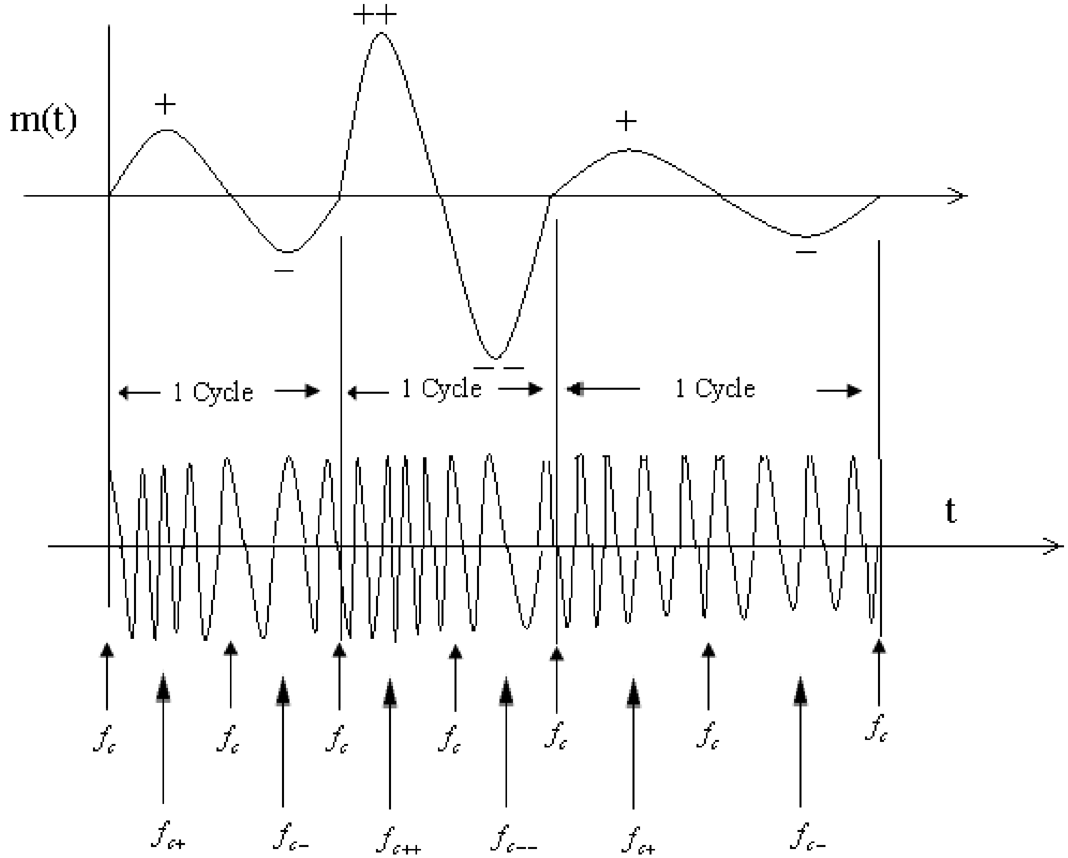


In general, $m(t)$ will be a 'band of signals', *i.e.* it will contain amplitude and fr variations. Both amplitude and frequency change in $m(t)$ at the input are trar (just) frequency changes in the FM output signal, *i.e.* the amplitude of the ou signal is constant.

Amplitude changes at the input are translated to deviation from the carrier at output. The larger the amplitude, the greater the deviation.

FM Signal Waveforms.

Frequency changes at the input are translated to rate of change of frequency output. An attempt to illustrate this is shown below:



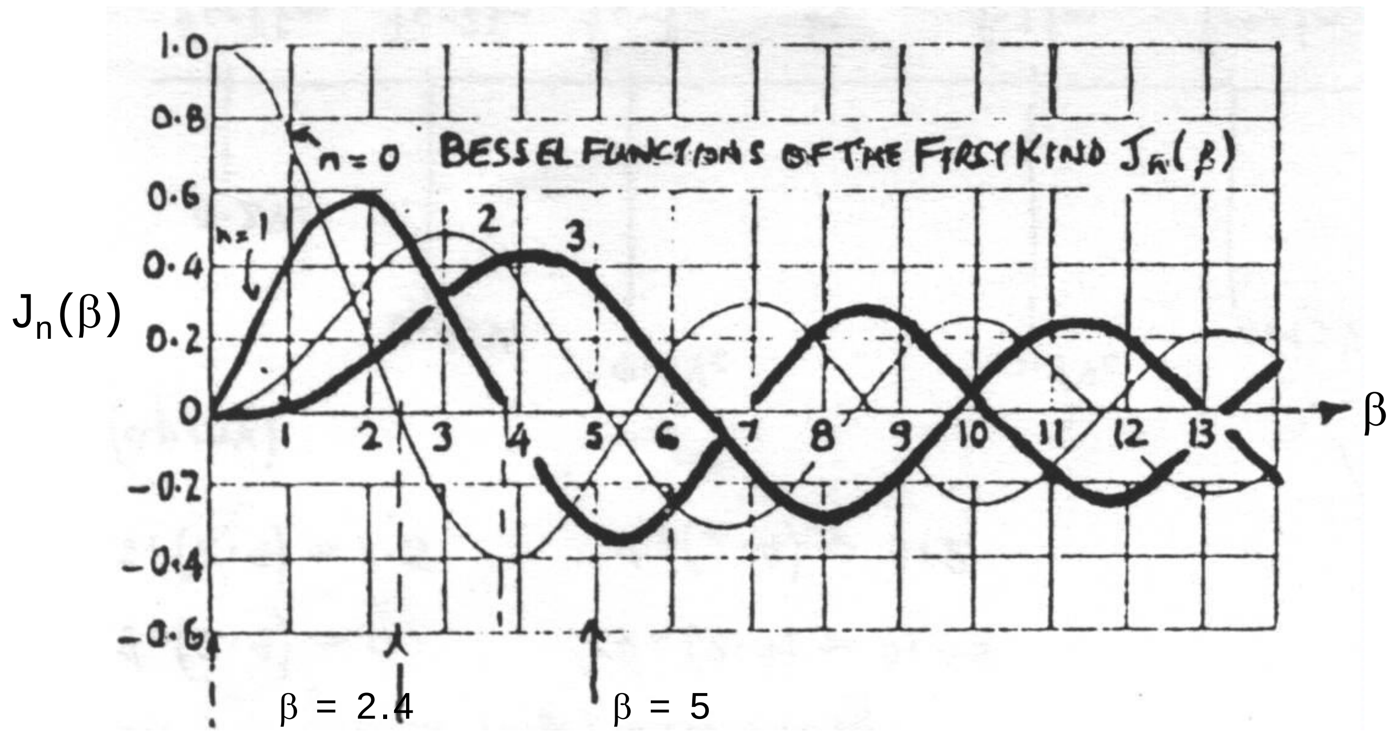
FM Spectrum – Bessel Coefficients.

The FM signal spectrum may be determined from

$$v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

The values for the Bessel coefficients, $J_n(\beta)$ may be found
graphs or, preferably, tables of ‘Bessel functions of the firs

FM Spectrum – Bessel Coefficients.

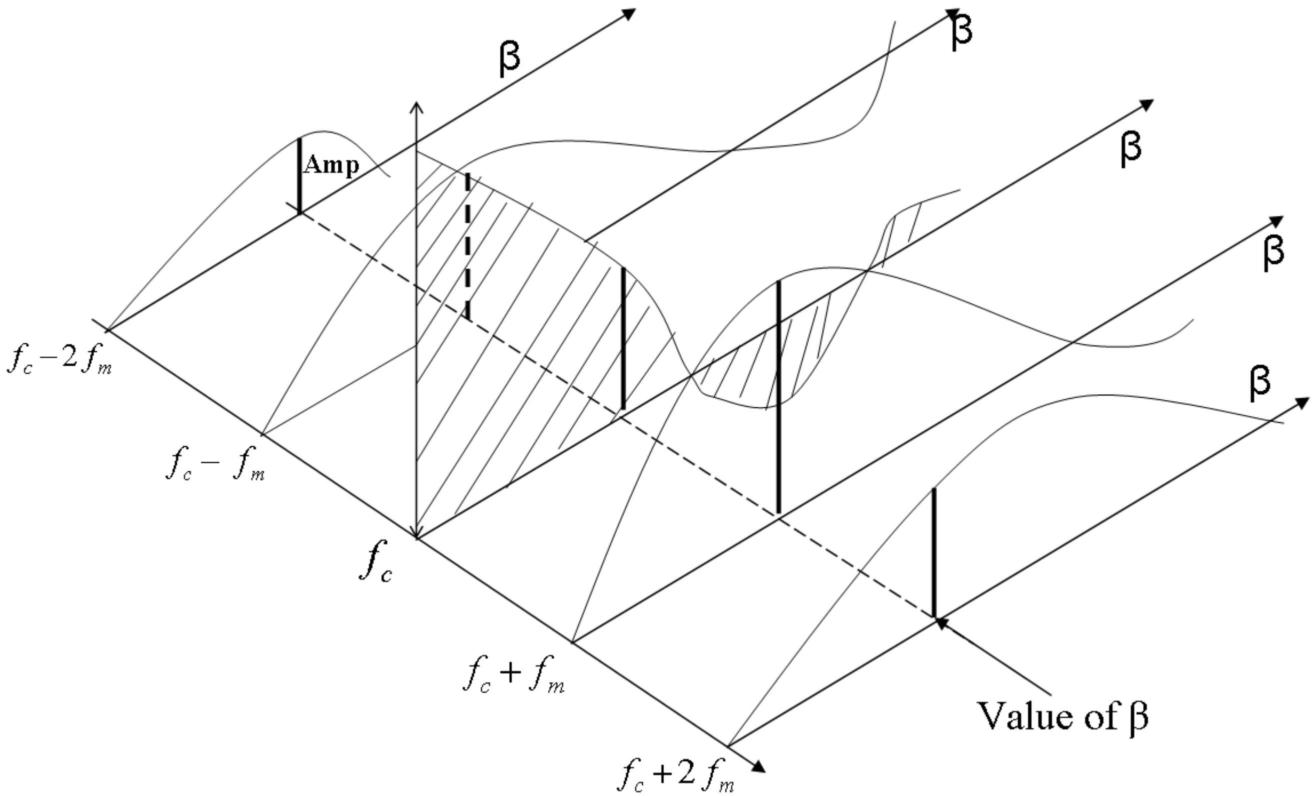


In the series for $v_s(t)$, $n = 0$ is the carrier component, i.e. $V_c J_0(\beta)\cos(\omega_c t)$, hence the $n = 0$ curve shows how the component at the carrier frequency, f_c, varies in a with modulation index β.

FM Spectrum – Bessel Coefficients.

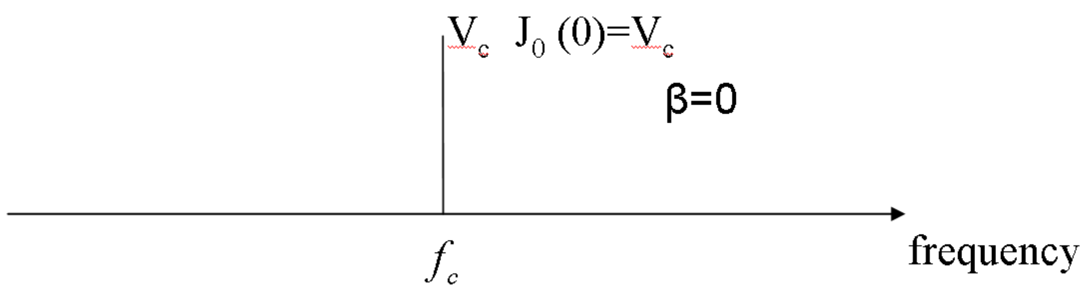
Hence for a given value of modulation index β, the values of $J_n(\beta)$ may be rea
graph and hence the component amplitudes ($V_c J_n(\beta)$) may be determined.

A further way to interpret these curves is to imagine them in 3 dimensions



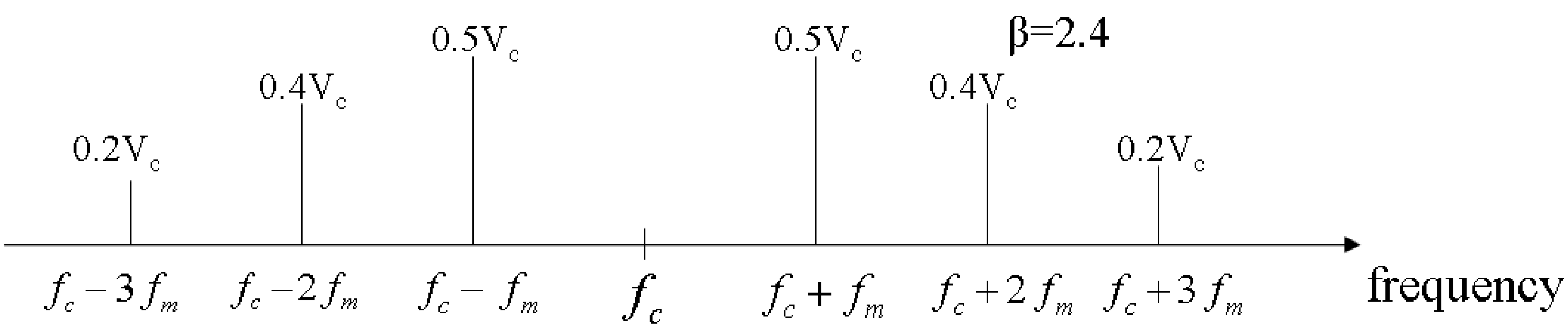
Examples from the graph

β = 0: When β **= 0 the carrier is unmodulated and J_0(0) = 1, all other J**



β = 2.4: From the graph (approximately)

$J_0(2.4) = 0$, $J_1(2.4) = 0.5$, $J_2(2.4) = 0.45$ and $J_3(2.4) = 0.2$



Significant Sidebands – Spectrum.

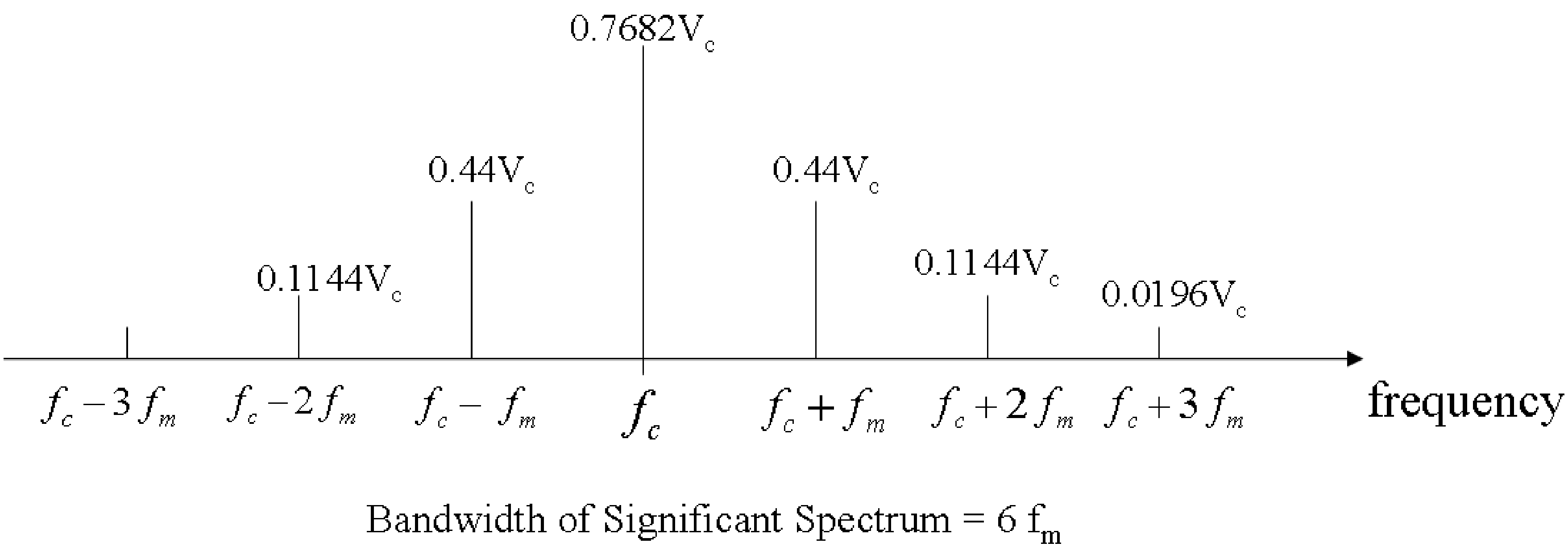
As may be seen from the table of Bessel functions, for values of n above a ce value, the values of $J_n(\beta)$ become progressively smaller. In FM the sidebands considered to be significant if $J_n(\beta) \geq 0.01$ (1%).

Although the bandwidth of an FM signal is infinite, components with amplitud $V_c J_n(\beta)$, for which $J_n(\beta) < 0.01$ are deemed to be insignificant and may be ign

Example: A message signal with a frequency f_m Hz modulates a carrier f_c to FM with a modulation index β = 1. Sketch the spectrum.

n	$J_n(1)$	Amplitude	Frequency
0	0.7652	$0.7652V_c$	f_c
1	0.4400	$0.44V_c$	f_c+f_m $f_c - f_m$
2	0.1149	$0.1149V_c$	f_c+2f_m $f_c - 2f_m$
3	0.0196	$0.0196V_c$	f_c+3f_m $f_c -3 f_m$
4	0.0025	*Insignificant*	
5	0.0002	*Insignificant*	

Significant Sidebands – Spectrum.



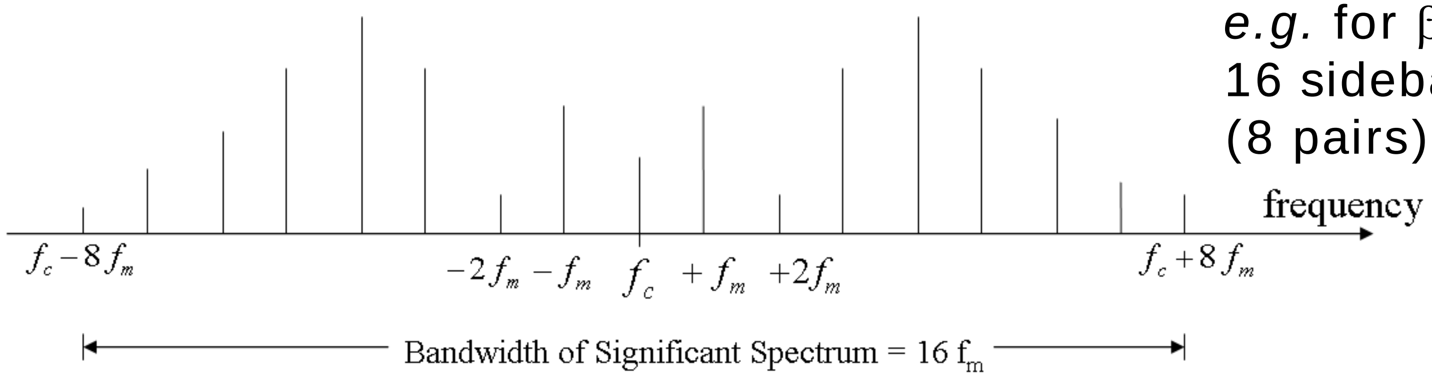
As shown, the bandwidth of the spectrum containing sign components is $6f_m$, for β = 1.

Significant Sidebands – Spectrum.

The table below shows the number of significant sidebands for various modu indices (β) and the associated spectral bandwidth.

β	No of sidebands ≥ 1% of unmodulated carrier	Bandwidth
0.1	2	$2f_m$
0.3	4	$4f_m$
0.5	4	$4f_m$
1.0	6	$6f_m$
2.0	8	$8f_m$
5.0	16	$16f_m$
10.0	28	$28f_m$

e.g. for β = 5, 16 sidebands (8 pairs).



Carson's Rule for FM Bandwidth.

An approximation for the bandwidth of an FM signal is giv
BW = 2(Maximum frequency deviation + highest modulat
frequency)

$$\text{Bandwidth} = 2(\Delta f_c + f_m)$$ **Carson's Rule**

Narrowband and Wideband FM

Narrowband FM NBFM

From the graph/table of Bessel functions it may be seen that for small β, ($\beta \leq$
there is only the carrier and 2 significant sidebands, *i.e.* BW = $2fm$.

FM with $\beta \leq 0.3$ is referred to as **narrowband FM** (NBFM) (Note, the bandwi
the same as DSBAM).

Wideband FM WBFM

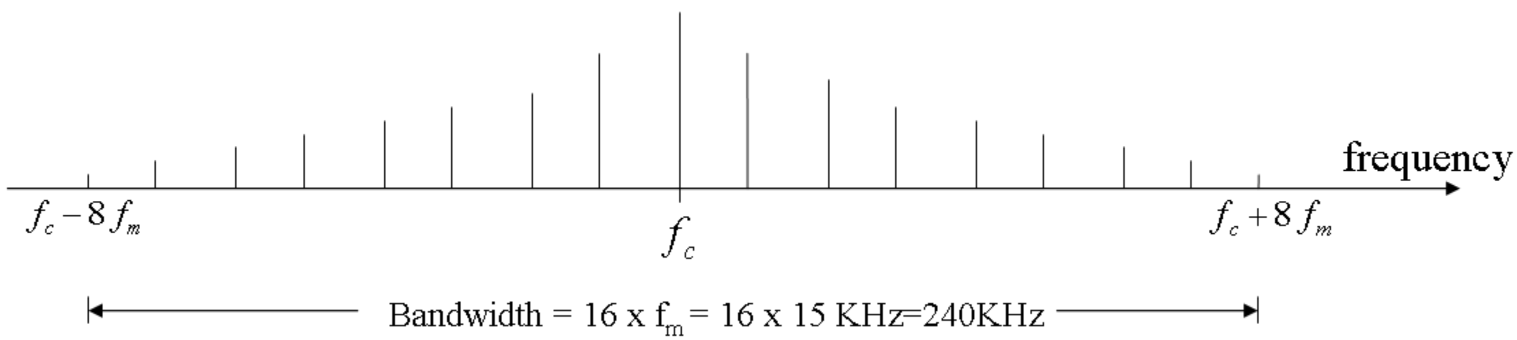
For $\beta > 0.3$ there are more than 2 significant sidebands. As β increases the n
sidebands increases. This is referred to as **wideband FM** (WBFM).

VHF/FM

VHF/FM (Very High Frequency band = 30MHz – 300MHz) radio transmissions band 88MHz to 108MHz have the following parameters:

Max frequency input (*e.g.* music)	15kHz	f_m
Deviation	75kHz	$\Delta f_c = \alpha V_m$
Modulation Index β	5	$\beta = \dfrac{\Delta f_c}{f_m}$

For β = 5 there are 16 sidebands and the FM signal bandwidth is 16*fm* = 16 x = 240kHz. Applying Carson's Rule BW = 2(75+15) = 180kHz.



Comments FM

- The FM spectrum contains a carrier component and an infinite number of si at frequencies $f_c \pm nf_m$ (n = 0, 1, 2, ...)

$$\text{FM signal,} \; v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

- In FM we refer to sideband pairs <u>not</u> upper and lower sidebands. Carrier or components <u>may not</u> be suppressed in FM.

- The relative amplitudes of components in FM depend on the values of $J_n(\beta)$, where $\beta = \dfrac{\alpha V_m}{f_m}$ thus the component at the carrier frequency depends on $m(t)$, as d other components and none may be suppressed.

Comments FM

- Components are significant if $J_n(\beta) \geq 0.01$. For $\beta << 1$ ($\beta \approx 0.3$ or less) only J $J_1(\beta)$ are significant, *i.e.* only a carrier and 2 sidebands. Bandwidth is $2f_m$, s DSBAM in terms of bandwidth - called NBFM.

- Large modulation index $\beta = \frac{\Delta f_c}{f_m}$ means that a large bandwidth is required – ca WBFM.

- The FM process is non-linear. The principle of superposition does not apply. $m(t)$ is a band of signals, *e.g.* speech or music the analysis is very difficult (impossible?). Calculations usually assume a single tone frequency equal to maximum input frequency. *E.g.* $m(t) \equiv$ band 20Hz $\rightarrow$ 15kHz, fm = 15kHz is

Power in FM Signals.

From the equation for FM $v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$

we see that the peak value of the components is $V_c J_n(\beta)$ for the n^{th} compone

Single normalised average power $\left(\frac{V_{pk}}{\sqrt{2}}\right)^2 = (V_{RMS})^2$ then the n^{th} component is

$$\left(\frac{V_c J_n(\beta)}{\sqrt{2}}\right)^2 = \frac{(V_c J_n(\beta))^2}{2}$$

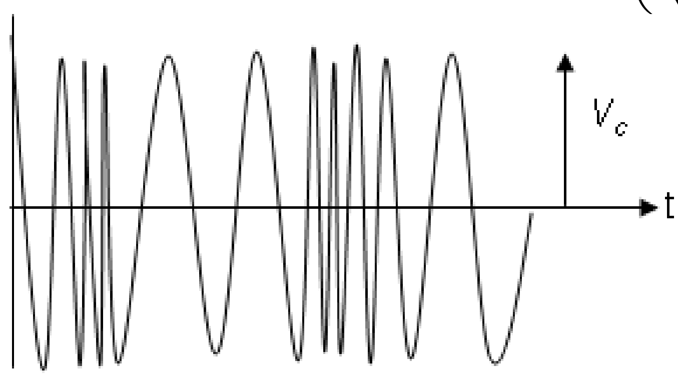
Hence, the total power in the infinite spectrum is

Total power $P_T = \sum_{n=-\infty}^{\infty} \frac{(V_c J_n(\beta))^2}{2}$

Power in FM Signals.

By this method we would need to carry out an infinite number of calculations P_T. But, considering the waveform, the peak value is V_{-c}, which is constant.

Since we know that the RMS value of a sine wave is $\left(\frac{V_{pk}}{\sqrt{2}}\right)^2 = \frac{V_c}{\sqrt{2}}$

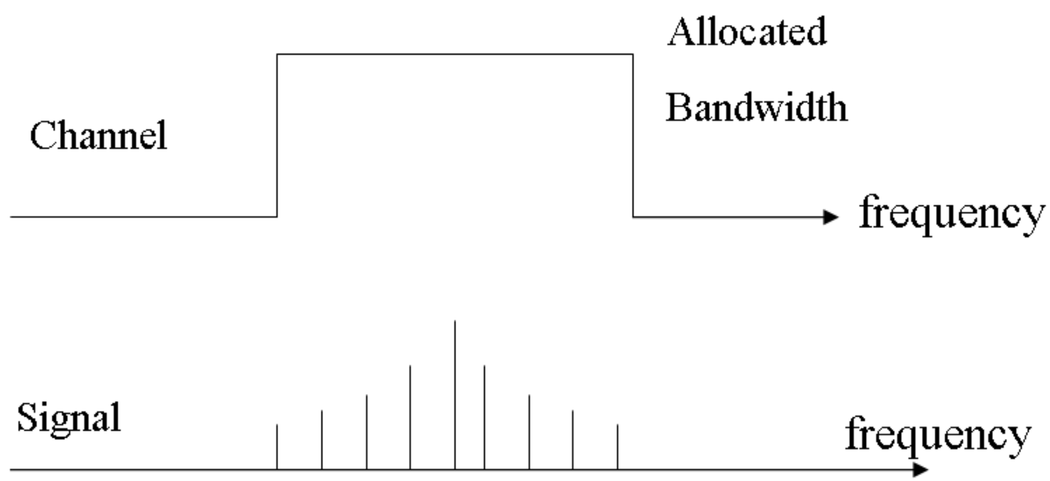


and power = $(V_{RMS})^2$ then we may deduce that $P_T = \left(\frac{V_c}{\sqrt{2}}\right)^2 = \frac{V_c^2}{2} = \sum_{n=-\infty}^{\infty} \frac{(V_c J_n(\beta))^2}{2}$

Hence, if we know V_c for the FM signal, we can find the total power P_T for the spectrum with a simple calculation.

Power in FM Signals.

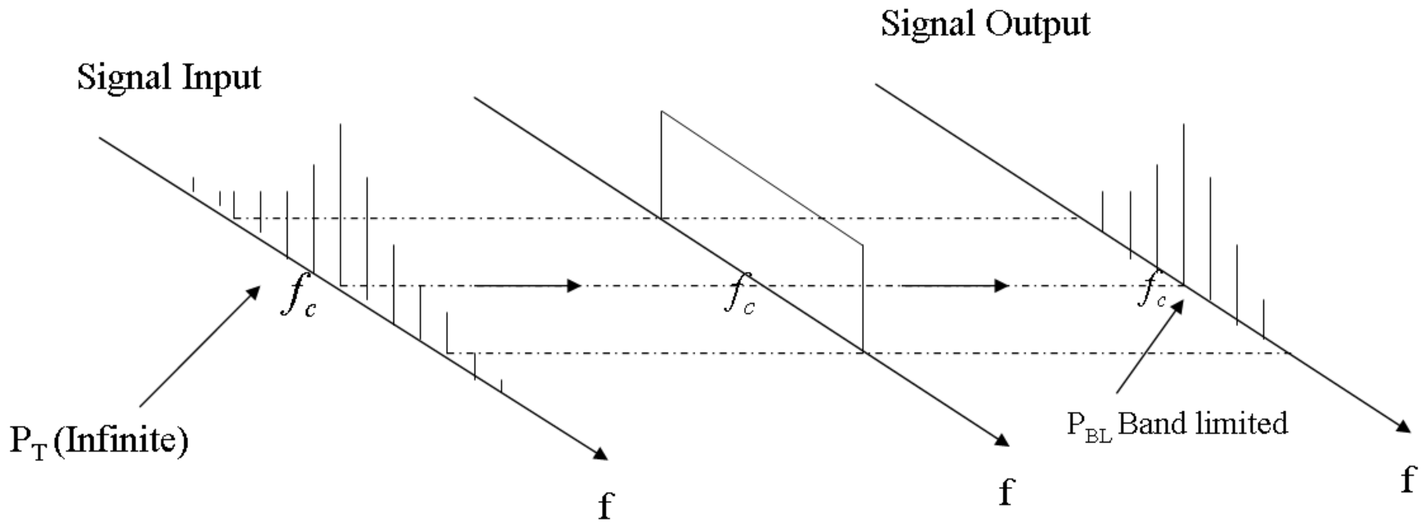
Now consider – if we generate an FM signal, it will contain an infinite number sidebands. However, if we wish to transfer this signal, *e.g.* over a radio or cal implies that we require an infinite bandwidth channel. Even if there was an ir channel bandwidth it would not all be allocated to one user. Only a limited ba is available for any particular signal. Thus we have to make the signal spectru the available channel bandwidth. We can think of the signal spectrum as a 'tr the channel bandwidth as a tunnel – obviously we make the train slightly less than the tunnel if we can.



Power in FM Signals.

However, many signals (*e.g.* FM, square waves, digital signals) contain an inf
number of components. If we transfer such a signal via a limited channel ban
we will lose some of the components and the output signal will be distorted.
an infinitely wide train through a tunnel, the train would come out distorted,
question is how much distortion can be tolerated?

Generally speaking, spectral components decrease in amplitude as we move
from the spectrum 'centre'.



Power in FM Signals.

In general distortion may be defined as

$$D = \frac{\text{Power in total spectrum - Power in Bandlimited spectrum}}{\text{Power in total spectrum}}$$

$$D = \frac{P_T - P_{BL}}{P_T}$$

With reference to FM the minimum channel bandwidth required would be jus enough to pass the spectrum of significant components. For a bandlimited FM spectrum, let *a* = the number of sideband pairs, *e.g.* for β = 5, *a* = 8 pairs (16 components). Hence, power in the bandlimited spectrum P_{BL} is

$$P_{BL} = \sum_{n=-a}^{a} \frac{(V_c J_n(\beta))^2}{2}$$ = carrier power + sideband powers.

Power in FM Signals.

Since $P_T = \frac{V_c^2}{2}$

Distortion $D = \frac{\frac{V_c^2}{2} - \frac{V_c^2}{2}\sum_{n=-a}^{a}(J_n(\beta))^2}{V_c^2/2} = 1 - \sum_{n=-a}^{a}(J_n(\beta))^2$

Also, it is easily seen that the ratio

$D = \frac{\text{Power in Bandlimited spectrum}}{\text{Power in total spectrum}} = \frac{P_{BL}}{P_T} = \sum_{n=-a}^{a}(J_n(\beta))^2$ = 1 – Distortion

i.e. proportion p_f power in bandlimited spectrum to total power = $\sum_{n=-a}^{a}(J_n(\beta))^2$

Example

Consider NBFM, with β = 0.2. Let V_c = 10 volts. The total power in the infinite spectrum $\frac{V_c^2}{2}$ = 50 Watts, *i.e.* $\sum_{n=-a}^{a}(J_n(\beta))^2$ = 50 Watts.

From the table – the significant components are

n	$J_n(0.2)$	Amp = $V_c J_n(0.2)$	Power = $\frac{(Amp)^2}{2}$
0	0.9900	9.90	49.005
1	0.0995	0.995	0.4950125
			P_{BL} = 49.5 Watts

i.e. the carrier + 2 sidebands contain $\frac{49.5}{50} = 0.99$ or 99% of the total power

Example

Distortion $= \dfrac{P_T - P_{BL}}{P_T} = \dfrac{50 - 49.5}{50} = 0.01$ or 1%.

Actually, we don't need to know V_c, *i.e.* alternatively

Distortion $= 1 - \sum_{n=-1}^{1} (J_n(0.2))^2$ (a = 1)

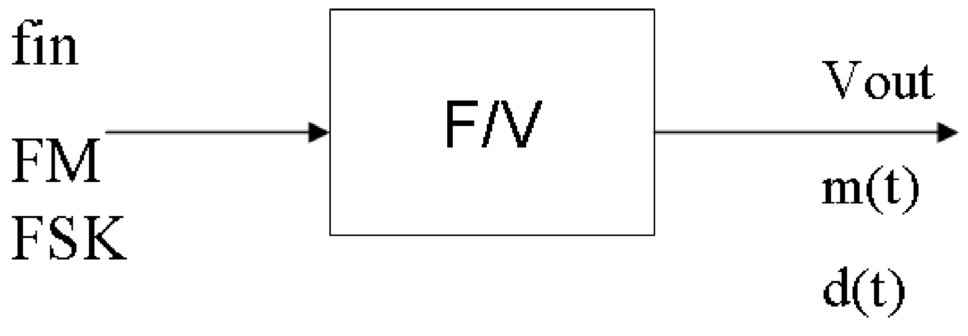
$$D = 1 - (0.99)^2 - (0.0995)^2 = 0.01$$

Ratio $\dfrac{P_{BL}}{P_T} = \sum_{n=-1}^{1} (J_n(\beta))^2 = 1 - D = 0.99$

FM Demodulation –General Principles.

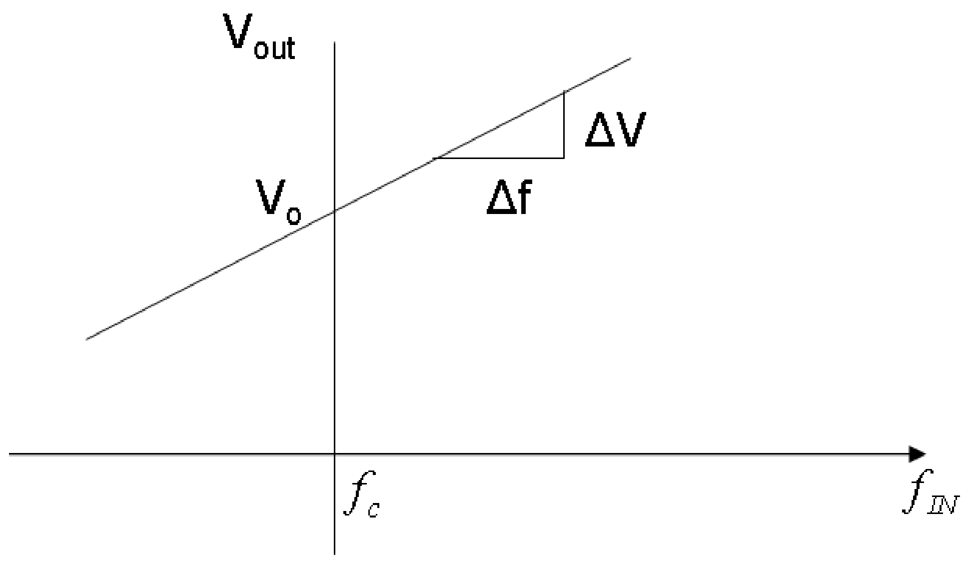
- An FM demodulator or frequency discriminator is essentially a frequency-to converter (F/V). An F/V converter may be realised in several ways, includin example, tuned circuits and envelope detectors, phase locked loops *etc.* Demodulators are also called FM discriminators.

- Before considering some specific types, the general concepts for FM demod will be presented. An F/V converter produces an output voltage, V_{OUT} which proportional to the frequency input, f_{IN}.



FM Demodulation –General Principles.

- If the input is FM, the output is $m(t)$, the analogue message signal. If the in the output is $d(t)$, the digital data sequence.
- In this case f_{IN} is the independent variable and V_{OUT} is the dependent variabl y axes respectively). The ideal characteristic is shown below.



We define V_o as the output when $f_{IN} = f_c$, the nominal input frequency.

FM Demodulation –General Principles.

The gradient $\frac{\Delta V}{\Delta f}$ is called the voltage conversion factor

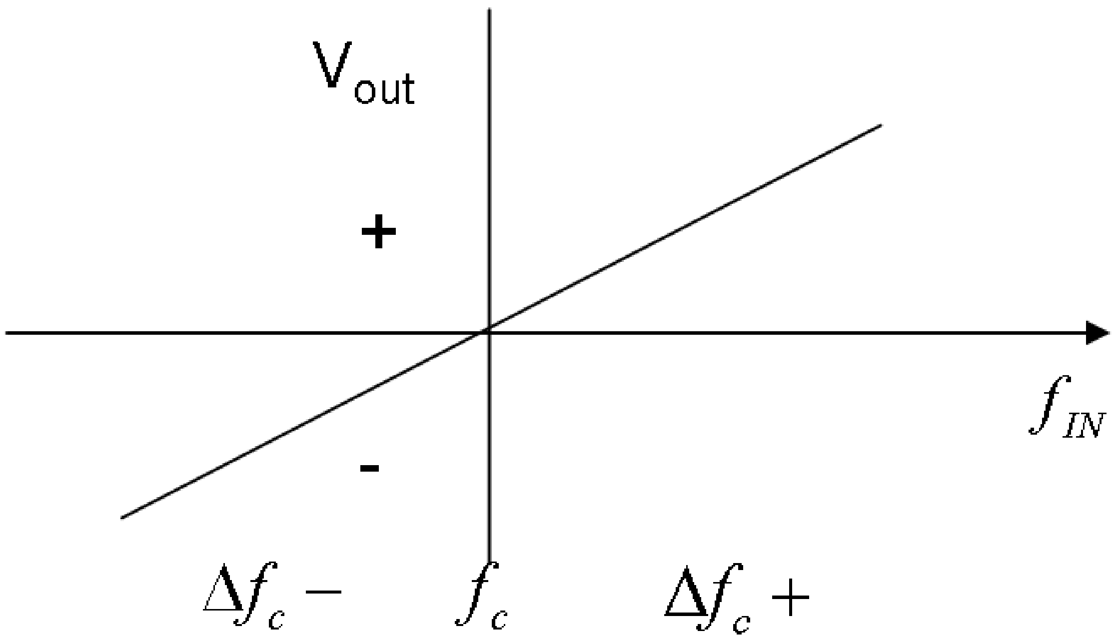
i.e. **Gradient = Voltage Conversion Factor**, *K* volts per Hz.

Considering *y = mx + c etc*. then we may say $V_{OUT} = V_0 + Kf_{IN}$ from the frequ
modulator, and since $V_0 = V_{OUT}$ when $f_{IN} = f_c$ then we may write

$$V_{OUT} = V_0 + K\alpha V_{IN}$$

where V_0 represents a DC offset in V_{OUT}. This DC offset may be removed by le
shifting or AC coupling, or the F/V may be designed with the characteristic sh

FM Demodulation –General Principles.



The important point is that $V_{OUT} = K\alpha V_{IN}$. If $V_{IN} = m(t)$ then the output conta message signal $m(t)$, and the FM signal has been demodulated.

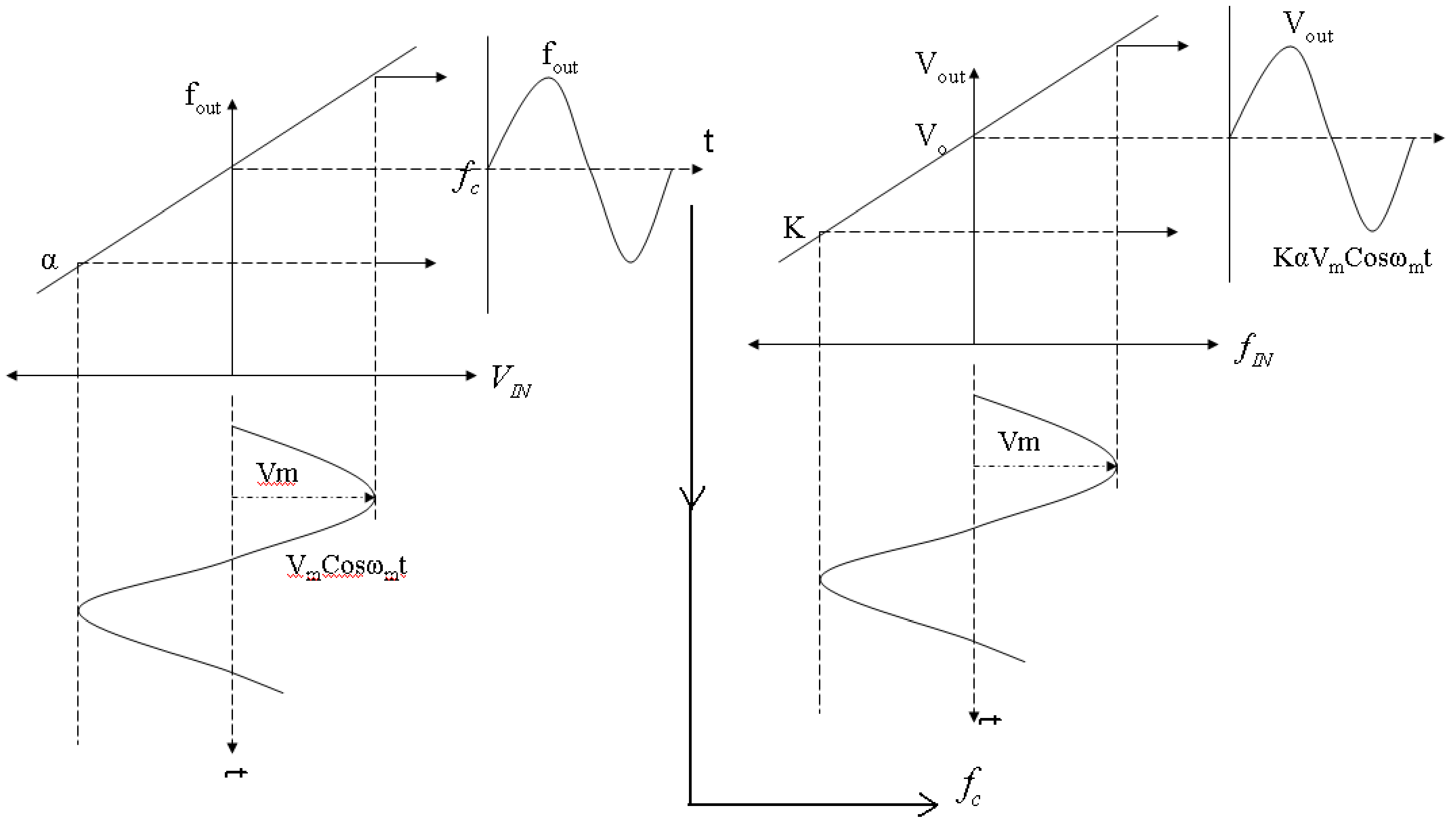
FM Demodulation –General Principles.

Often, but not always, a system designed so that $K = \frac{1}{\alpha}$, so that $K\alpha = 1$ and $V_{OUT} = m(t)$. A complete system is illustrated.

Gradient = α Hz/Volt
α = Frequency conversion factor
$f_{OUT} = f_c + \alpha V_{IN} = f_{IN}$
$f_{OUT} = f_c + \alpha m(t) = f_{IN}$

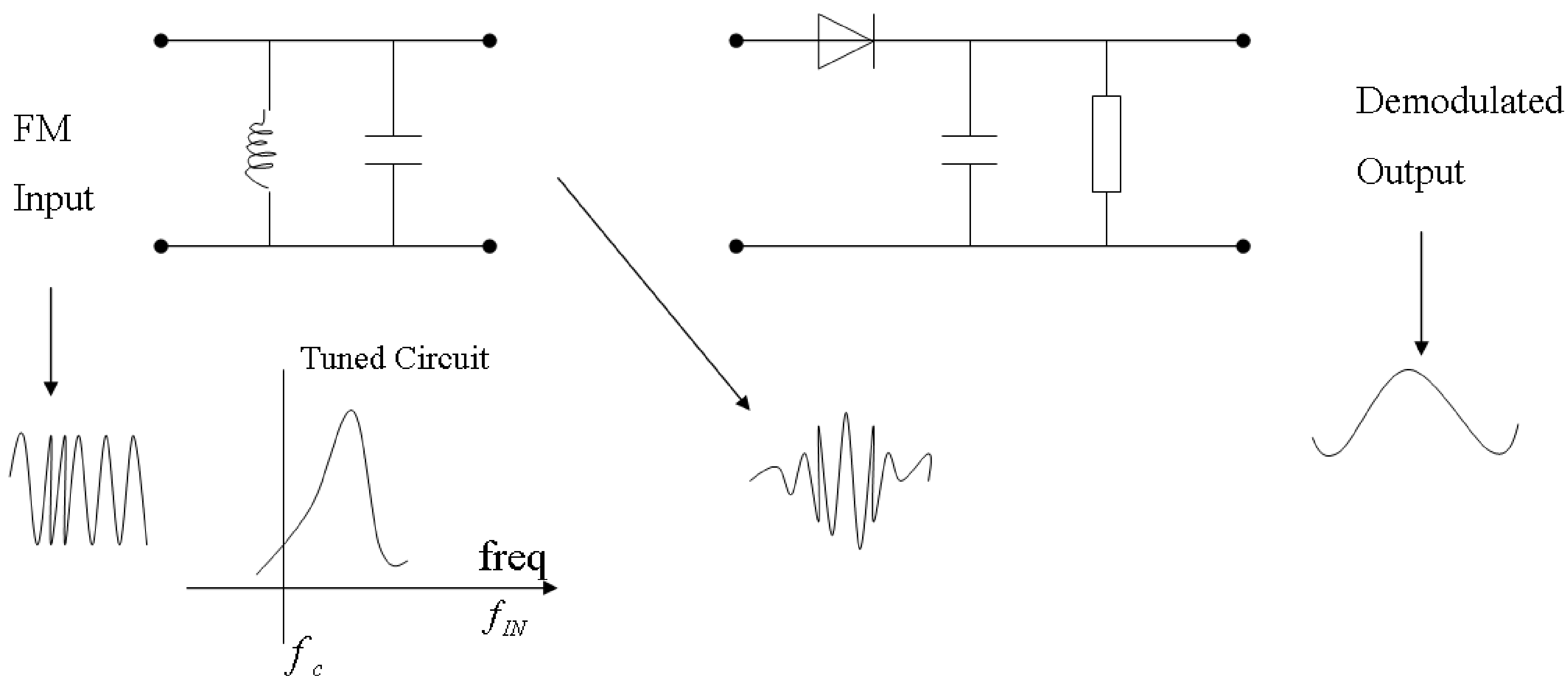
Gradient = K Hz/Volt
K = Voltage conversion factor
$V_{OUT} = V_0 + K\alpha V_{IN}$
$V_{OUT} = V_0 + K\alpha m(t)$

FM Demodulation –General Principles.



Methods

Tuned Circuit – One method (used in the early days of FM) is to use the slo tuned circuit in conjunction with an envelope detector.



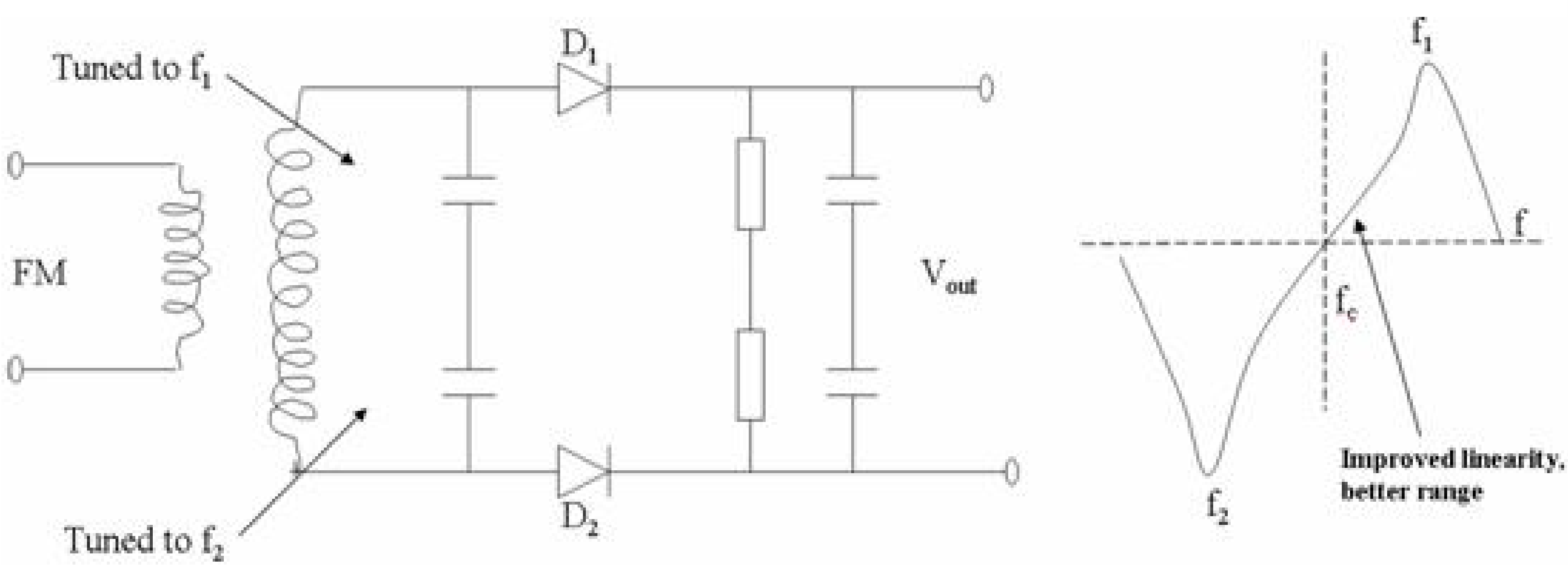
Methods

- The tuned circuit is tuned so the f_c, the nominal input frequency, is on the sl the centre of the tuned circuits. As the FM signal deviates about f_c on the tu slope, the amplitude of the output varies in proportion to the deviation from the FM signal is effectively converted to AM. This is then envelope detected b diode *etc* to recover the message signal.

- Note: In the early days, most radio links were AM (DSBAM). When FM came with its advantages, the links could not be changed to FM quickly. Hence, NB used (with a spectral bandwidth = $2fm$, *i.e.* the same as DSBAM). The carrie frequency *fc* was chosen and the IF filters were tuned so that *fc* fell on the sl filter response. Most FM links now are wideband with much better demodula

- A better method is to use 2 similar circuits, known as a **Foster-Seeley Disc**

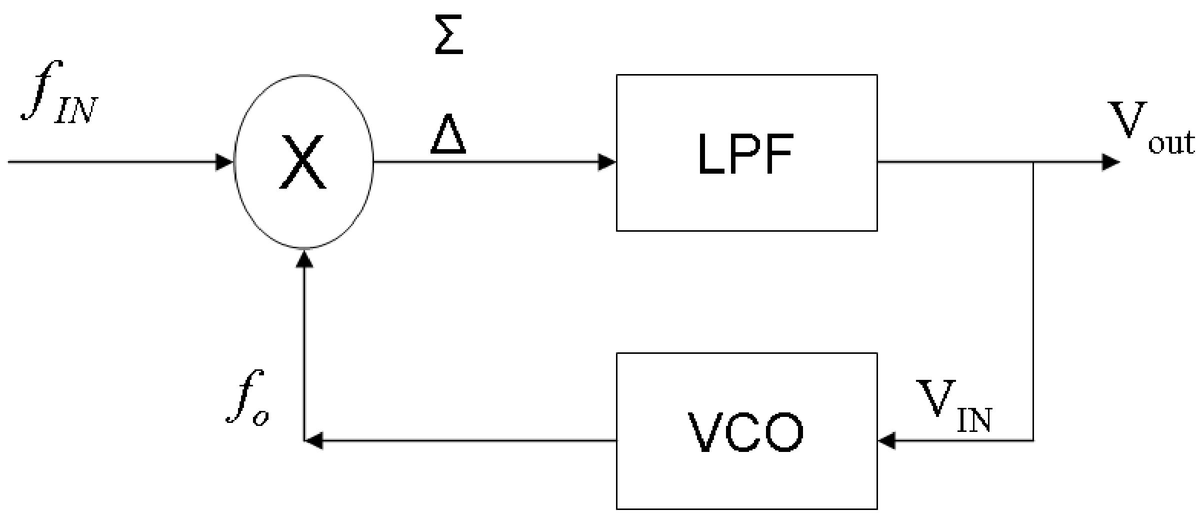
Foster-Seeley Discriminator



This gives the composite characteristics shown. Diode D_2 effectively inverts t tuned circuit response. This gives the characteristic *'S'* type detector.

Phase Locked Loops PLL

- A PLL is a closed loop system which may be used for FM demodulation. A fu analytical description is outside the scope of these notes. A brief descriptior presented. A block diagram for a PLL is shown below.



- Note the similarity with a synchronous demodulator. The loop comprises a m a low pass filter and VCO (V/F converter as used in a frequency modulator).

Phase Locked Loops PLL

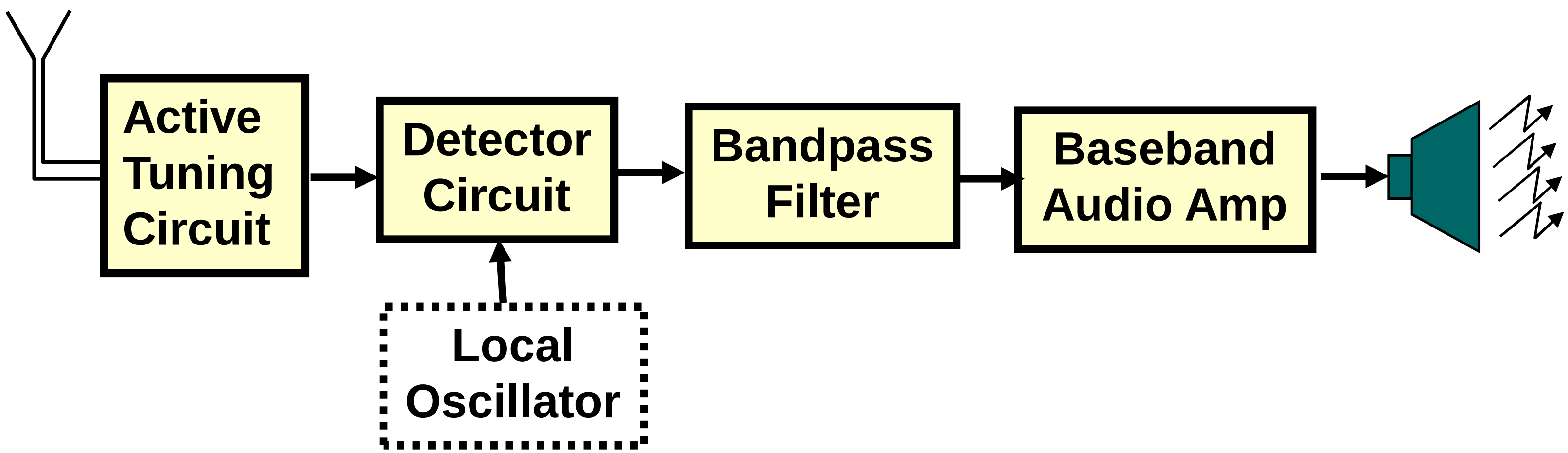
- The input f_{IN} is applied to the multiplier and multiplied with the VCO frequency output f_O, to produce $\Sigma = (f_{IN} + f_O)$ and $\Delta = (f_{IN} - f_O)$.
- The low pass filter passes only $(f_{IN} - f_O)$ to give *VOUT* which is proportional to $(f_{IN} - f_O)$.
- If $f_{IN} \approx f_O$ but not equal, $V_{OUT} = V_{IN}$, $\alpha f_{IN} - f_O$ is a low frequency (beat frequency) signal to the VCO.
- This signal, V_{IN}, causes the VCO output frequency f_O to vary and move towards f_{IN}.
- When $f_{IN} = f_O$, V_{IN} $(f_{IN} - f_O)$ is approximately constant (DC) and f_O is held constant, *i.e* locked to f_{IN}.
- As f_{IN} changes, due to deviation in FM, f_O tracks or follows f_{IN}. $V_{OUT} = V_{IN}$ changes to drive f_O to track f_{IN}.
- V_{OUT} is therefore proportional to the deviation and contains the message signal $m(t)$.

UNIT-III

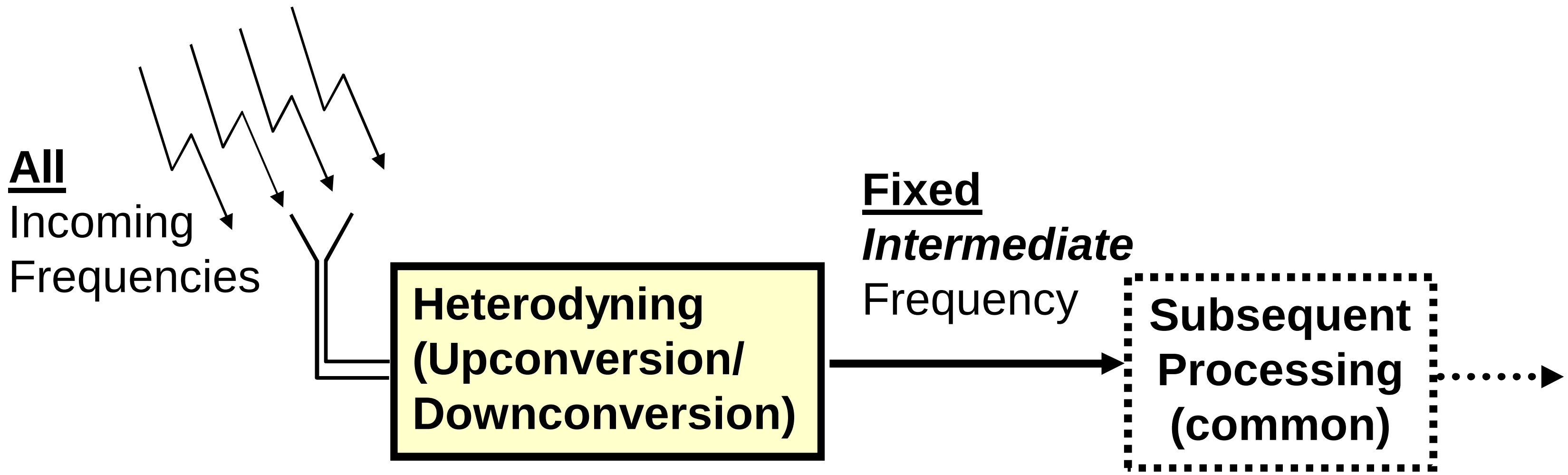
RADIO RECEIVERS

Tuned Radio Frequency (TRF) Receivers

- Composed of RF amplifiers and detectors.
- No frequency conversion. It is not often used.
- Difficult to design tunable RF stages.
- Difficult to obtain high gain RF amplifiers

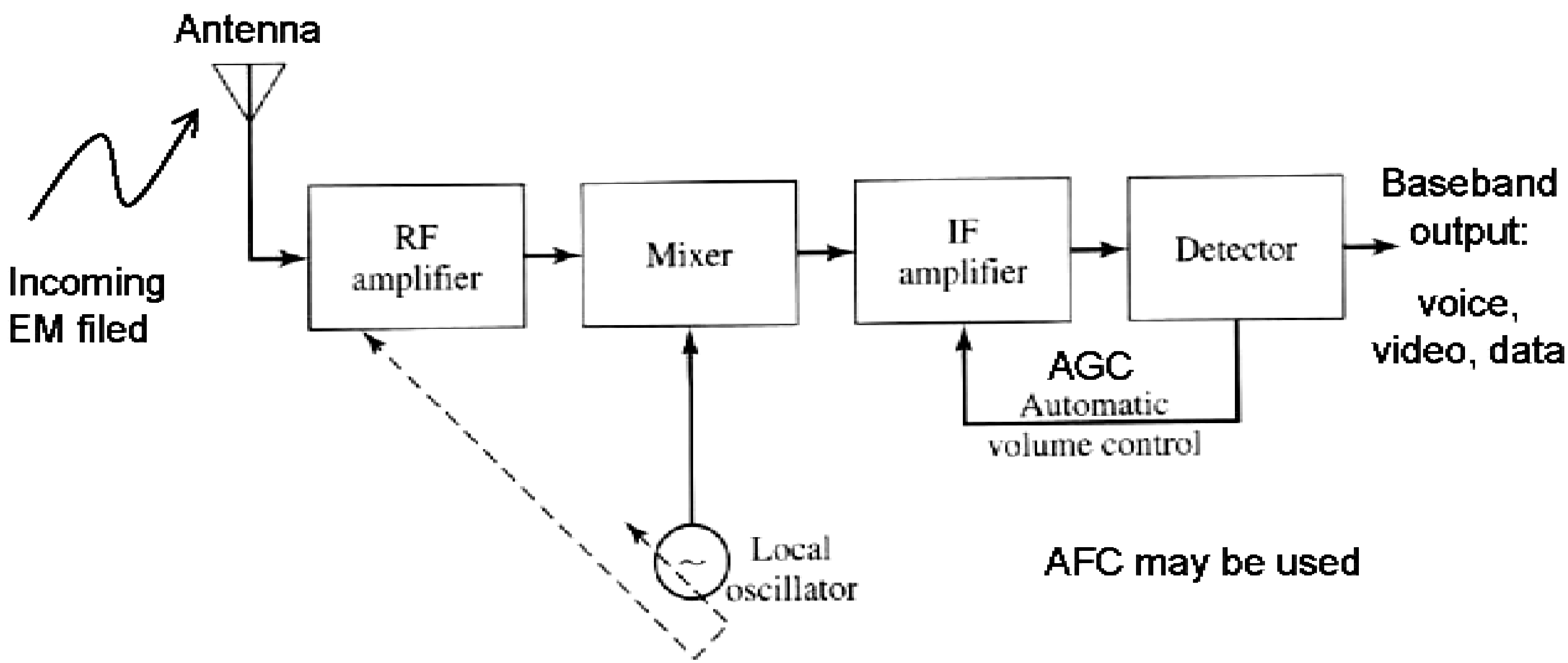


Heterodyning



Superheterodyne Receivers

- Most popular type of a radio receiver so far.
- Used for AM/FM & TV broadcasting, cellular & satellite systems, radars, GPS etc.
- Main idea: downconvert RF signal to some fixed lower (intermediate) frequency, then amplify it and detect.



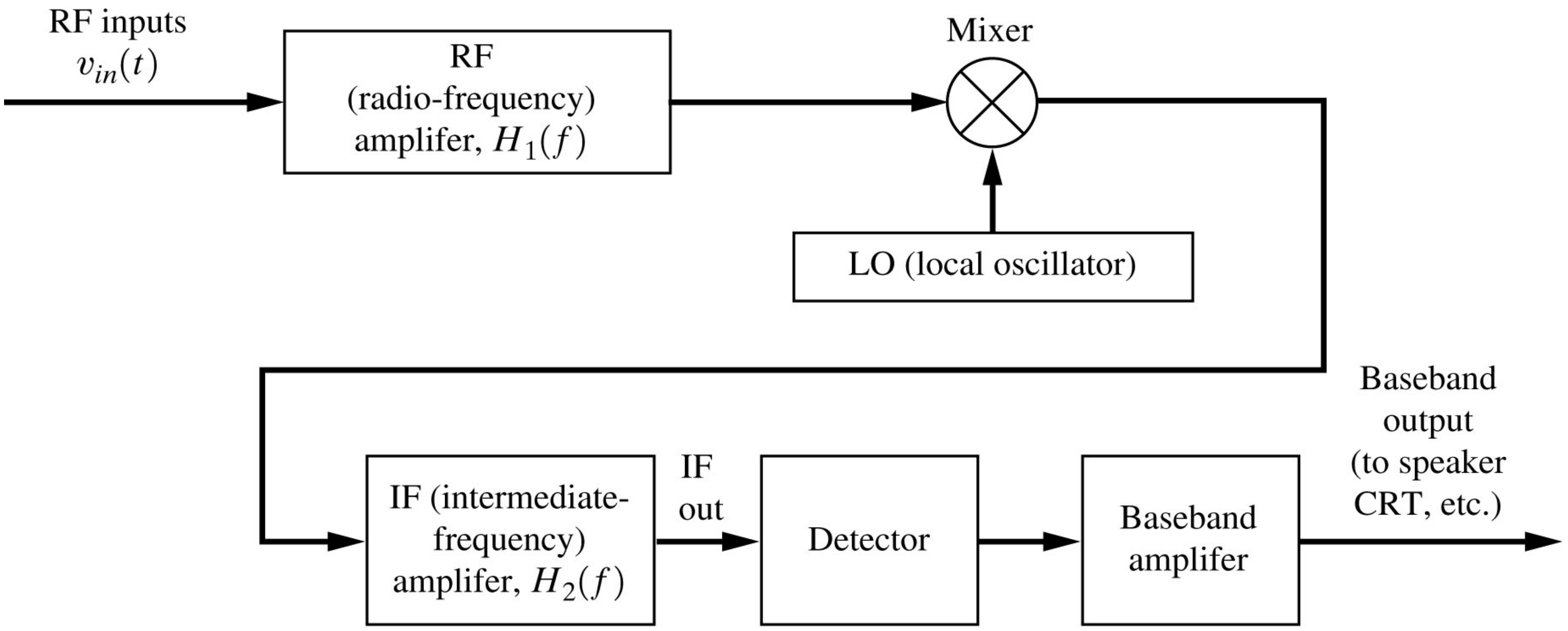
Superheterodyne Receiver Diagram

Superheterodyne Receiver

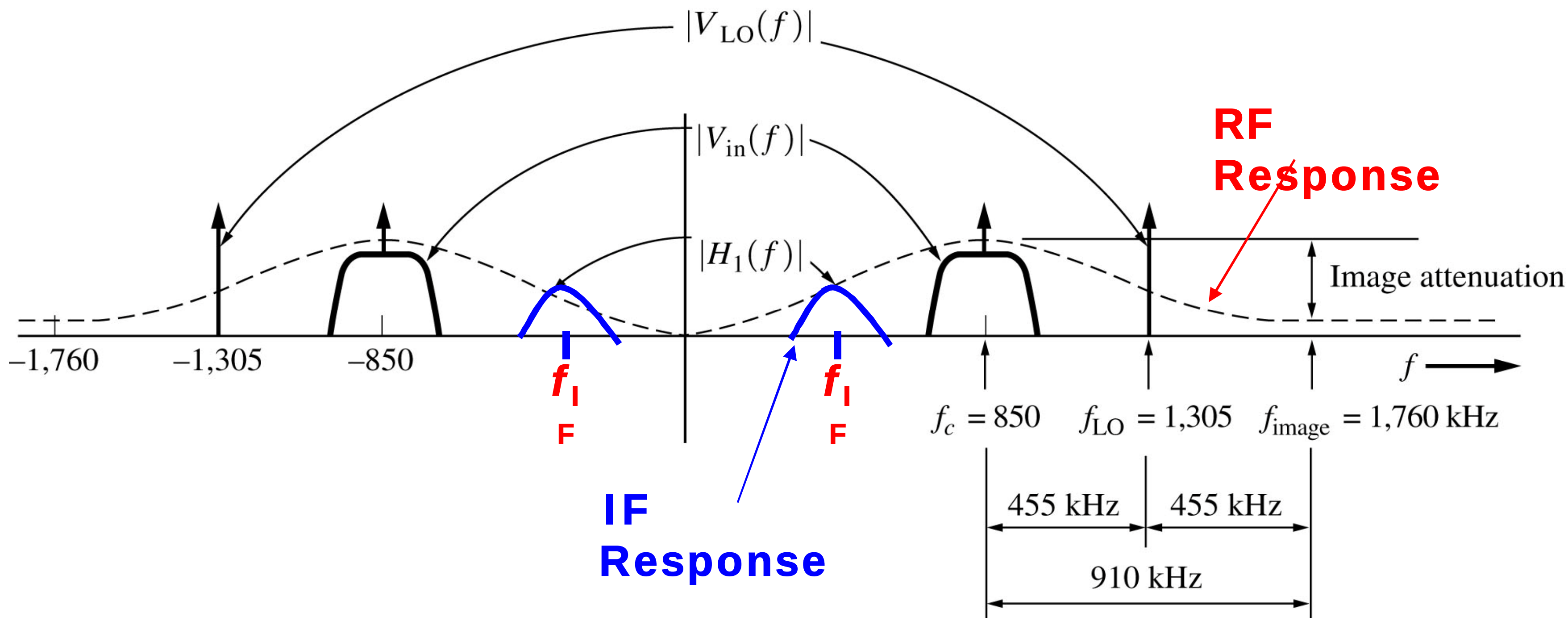
- RF amplifier: amplifies a weak RF signal coming out of the antenna. Rejects the image frequency. Bandwidth: much wider than the signal bandwidth.
- Mixer: together with the local oscillator downconverts the RF signal to the IF frequency band.
- IF amplifier: amplifies the IF signal significantly (up to 10^6) and rejects adjacent channel signals and interference (frequency selectivity). Its bandwidth is the same as the signal bandwidth.
- Detector (demodulator): demodulates (recovers) the message signal.
- AGC: adjusts the IF amplifier gain according to the signal level (to keep the average signal amplitude almost constant)
- Local oscillator: allows tuning the receiver to a desired channel (frequency).

Superheterodyne Receivers

➢ The RF and IF frequency responses $H_1(f)$ and $H_2(f)$ are important in providing the required reception characteristics.



Superheterodyne Receivers



Superheterodyne Receivers

TABLE 4–4 SOME POPULAR IF FREQUENCIES IN THE UNITED STATES.

IF Frequency	Application
262.5 kHz	AM broadcast radios (in auto
455 kHz	AM broadcast radios
10.7 MHz	FM broadcast radios
21.4 MHz	FM two-way radios
30 MHz	Radar receivers
43.75 MHz (video carrier)	TV sets
60 MHz	Radar receivers
70 MHz	Satellite receivers

Superheterodyne Receiver Frequencies

- IF must be such that the image response is rejected by RF amplifier.

up-side conversion

$$f_{LO} = f_c + f_{IF}$$

down-side conversion

$$f_{LO} = f_c - f_{IF}$$

- Image response: the same effect as that of the desired signal -> must be rejected!
- Image frequency:

up-side conversion

$$f_{image} = f_{LO} + f_{IF} = \\ = f_c + 2f_{IF}$$

down-side conversion

$$f_{image} = f_{LO} - f_{IF} = \\ f_c - 2f_{IF}$$

image rejection ->

$$\Delta f_{RF} < 2f_{IF}$$

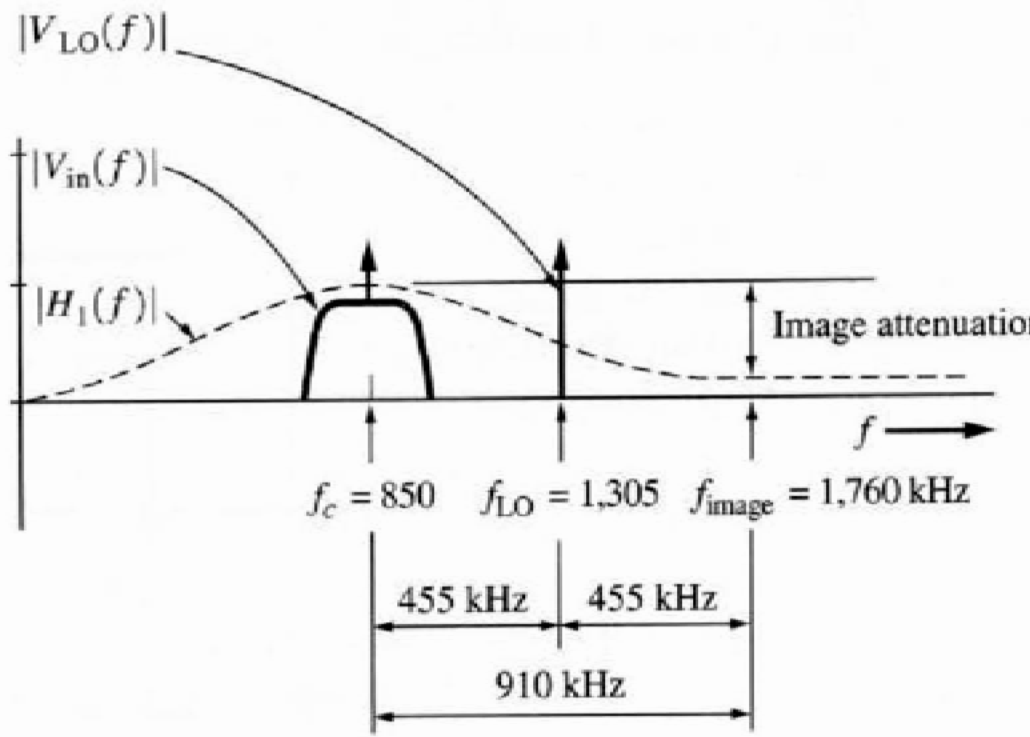
Superheterodyne Receiver Frequencies

- Incoming RF signal: $f_c = 850\ \text{kHz}$ IF signal: $f_{IF} = 455\ \text{kHz}$
- Up-side conversion: $f_{LO} = f_c + f_{IF} = 1305\ \text{kHz}$
- Image frequency: $f_{image} = f_{LO} + f_{IF} = f_c + 2 f_{IF} = 1760\ \text{kHz}$

Note: image rejection is due to RF amplifier only!

IF must be high enough to reject the image response.

On the other hand, IF must be low enough to provide large gain and adjacent channel rejection.



Frequency Conversion Process

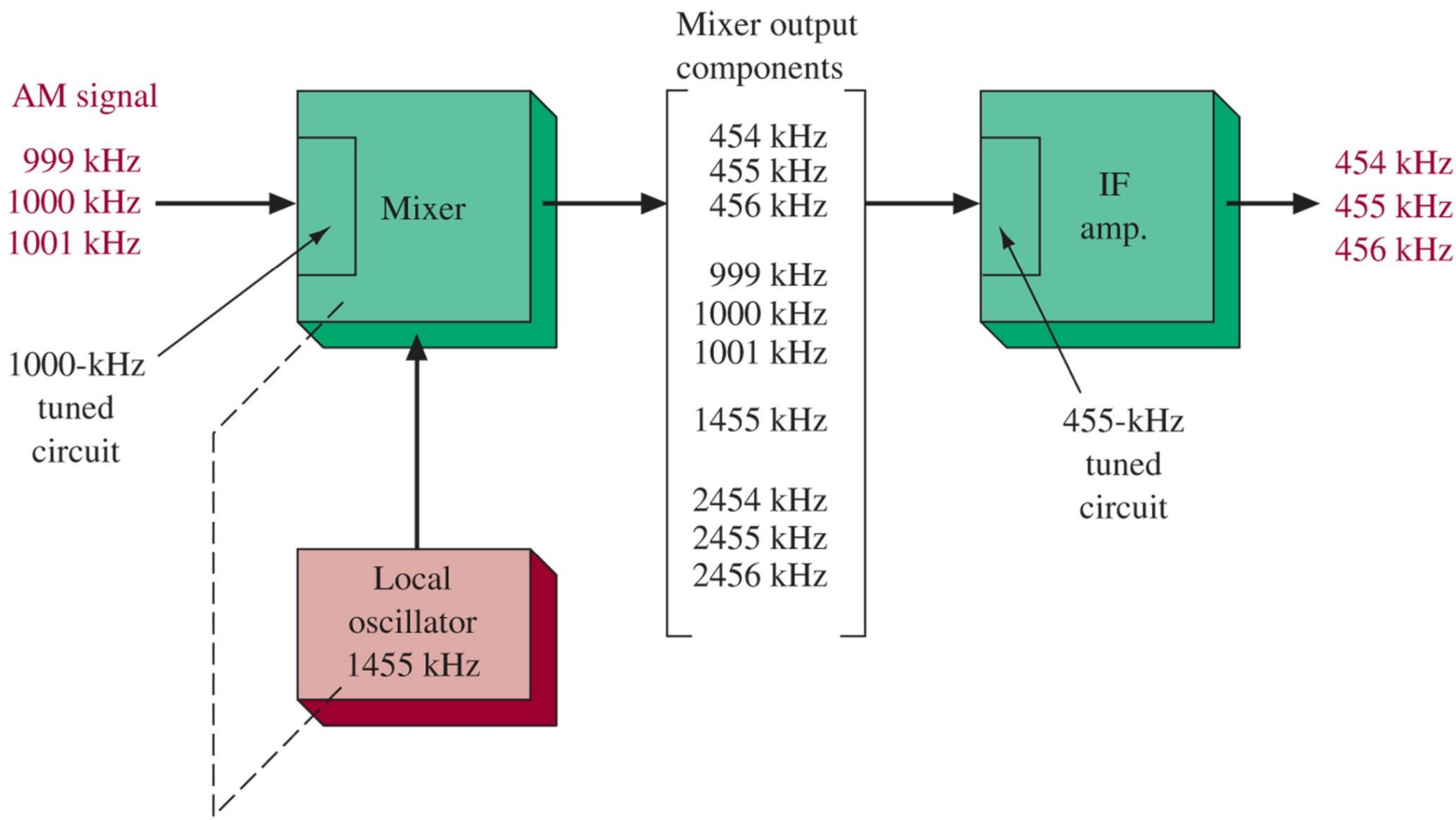


Image Frequencies

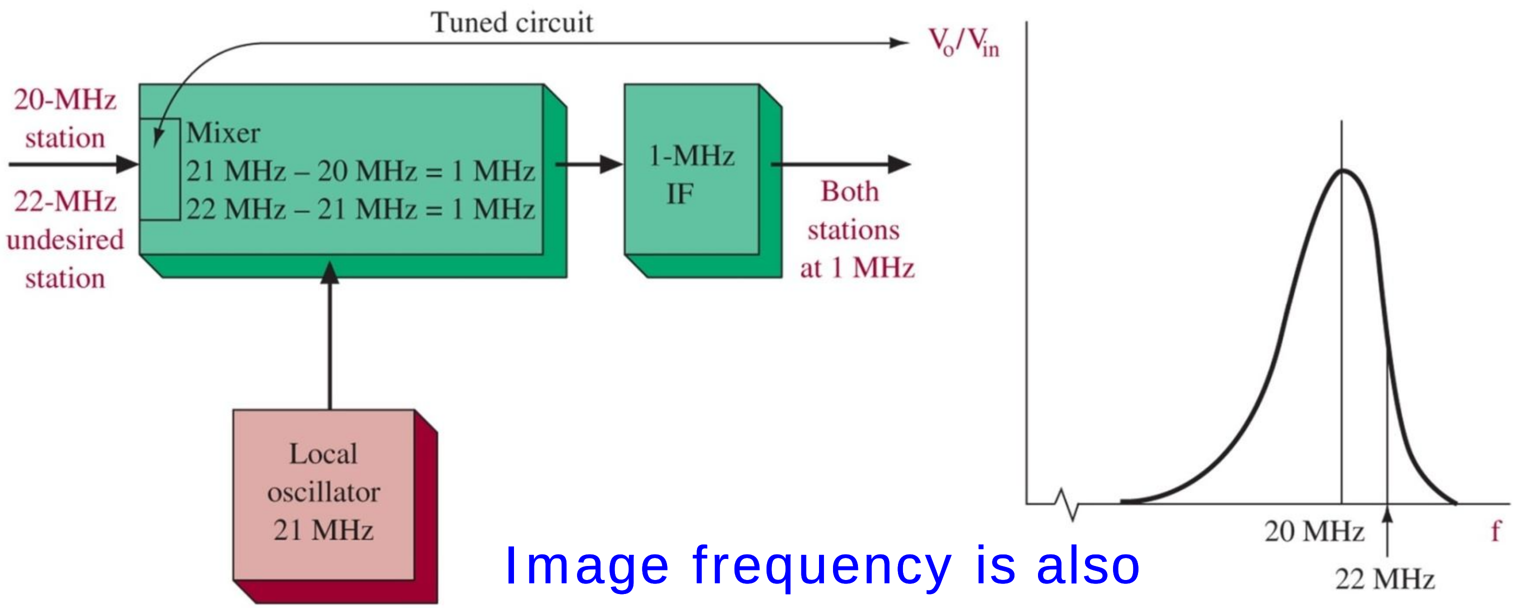


Image frequency is also received

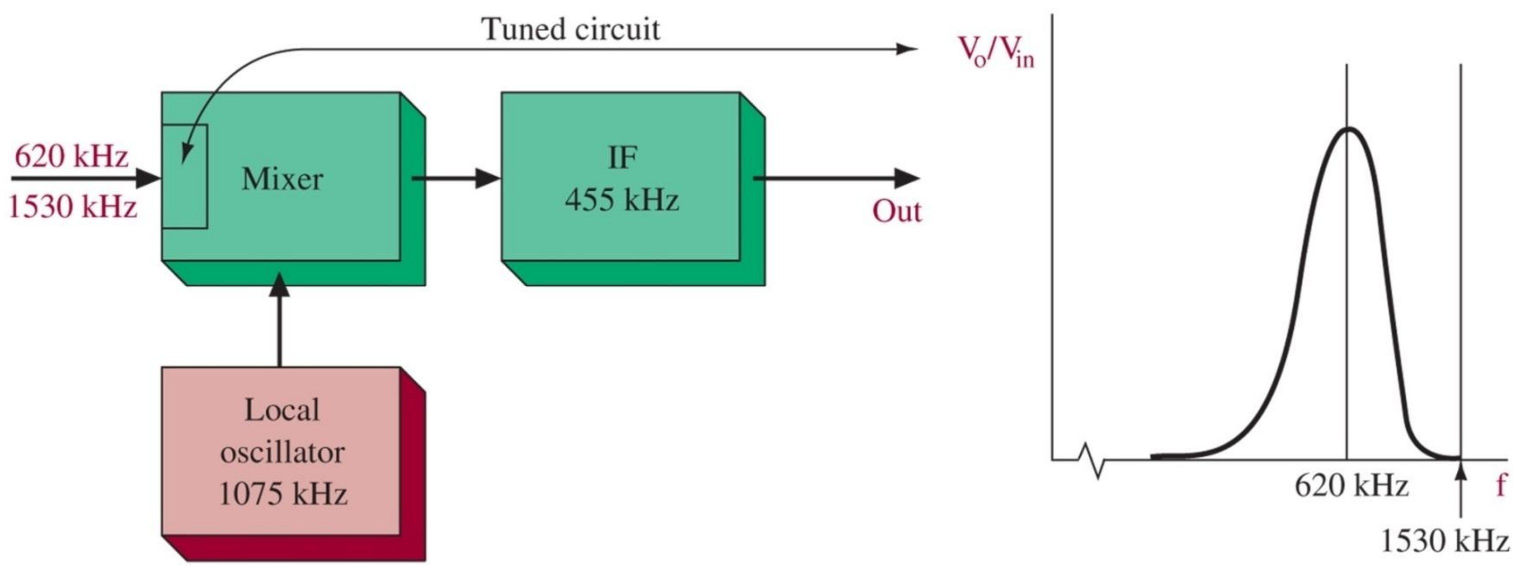
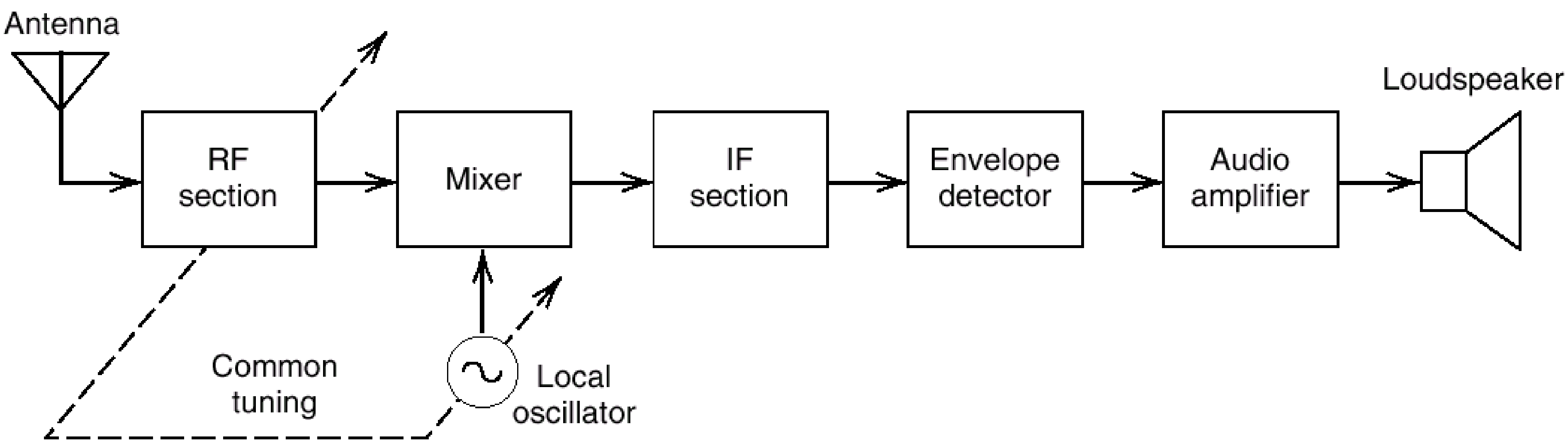
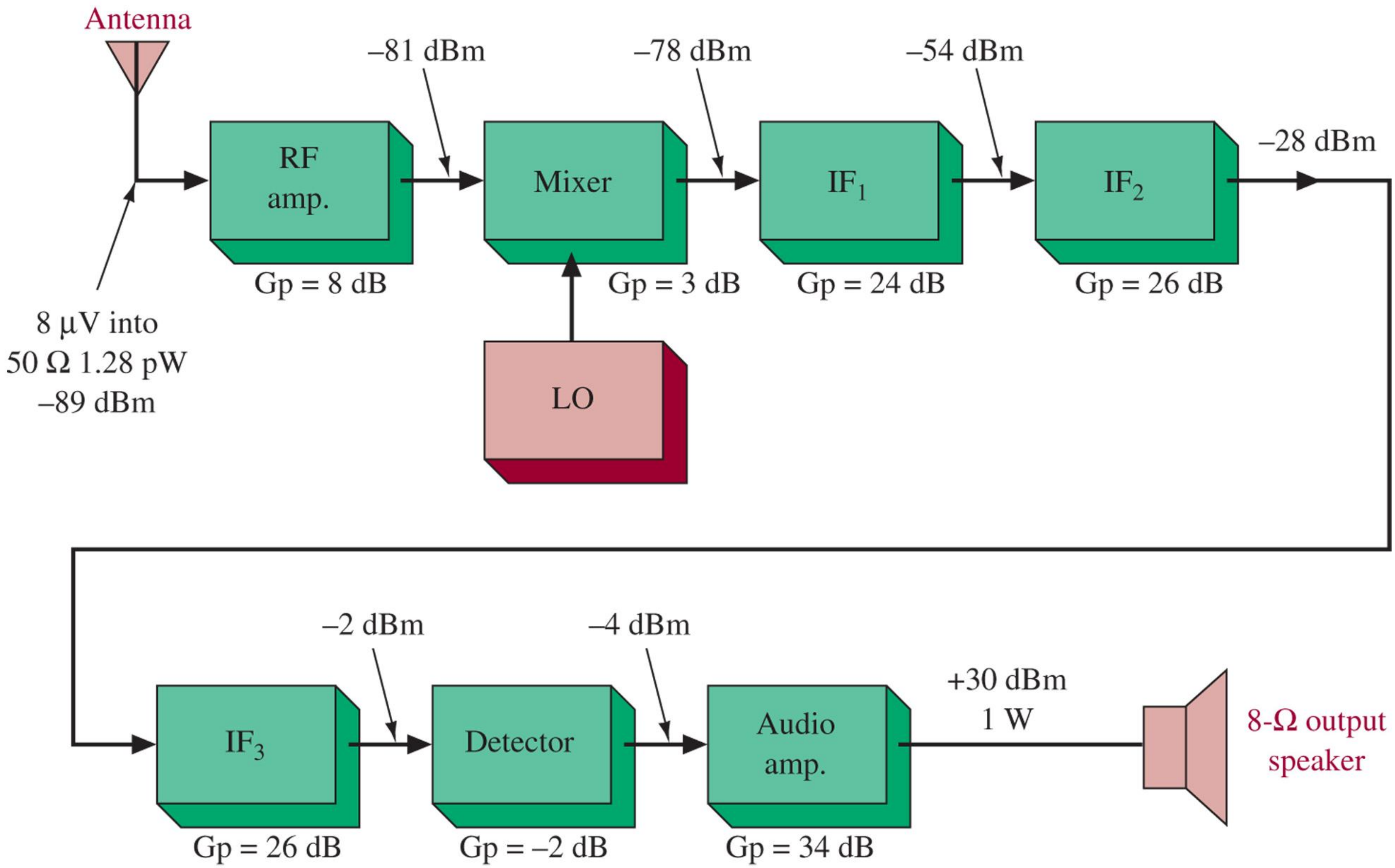


Image frequency not a problem.

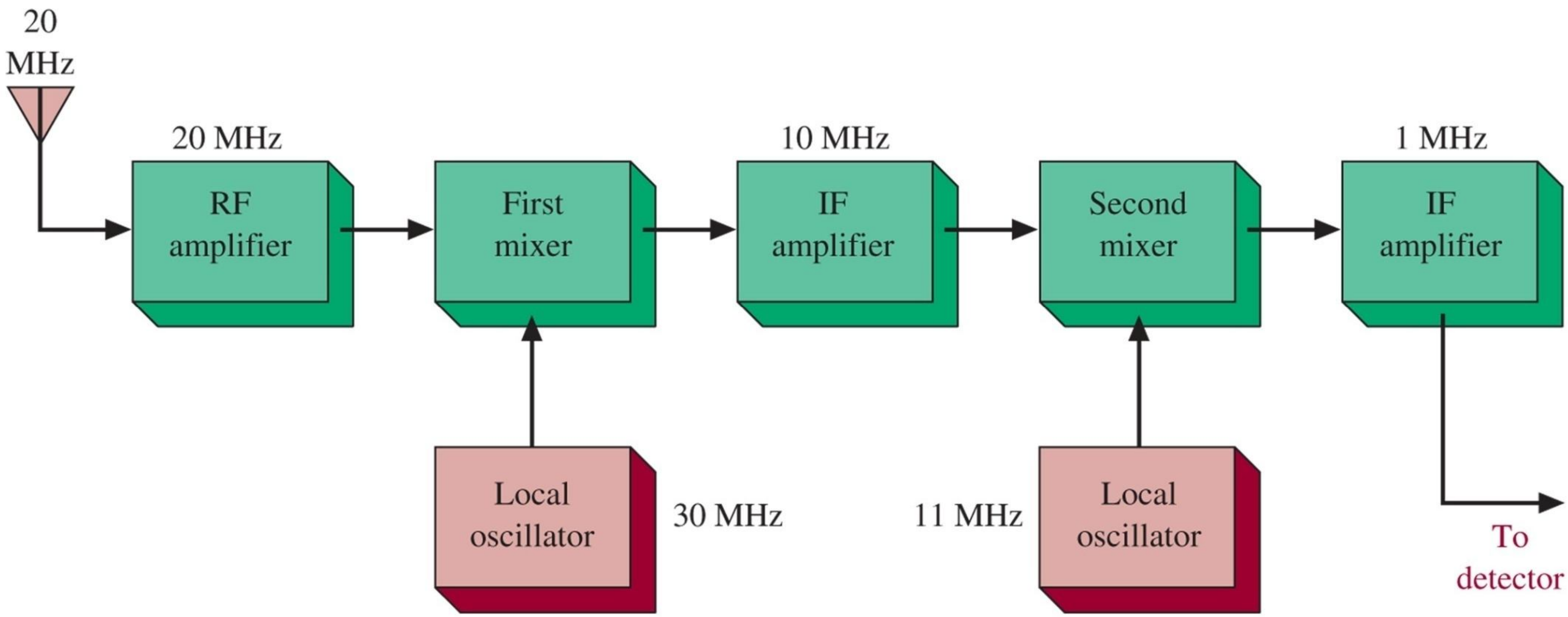
AM Radio Receiver



Superheterodyne Receiver Typical Signal Levels



Double-conversion block diagram.



Noise in Communication Systems

Noise in Communication Systems

1. Introduction
2. Thermal Noise
3. Shot Noise
4. Low Frequency or Flicker Noise
5. Excess Resister Noise
6. Burst or Popcorn Noise
7. General Comments
8. Noise Evaluation – Overview
9. Analysis of Noise in Communication Systems
   - Thermal Noise
   - Noise Voltage Spectral Density
   - Resistors in Series
   - Resistors in Parallel
10. Matched Communication Systems
11. Signal - to – Noise
12. Noise Factor – Noise Figure
13. Noise Figure / Factor for Active Elements
14. Noise Temperature
15. Noise Figure / Factors for Passive Elements
16. Review – Noise Factor / Figure / Temperature
17. Cascaded Networks
18. System Noise Figure
19. System Noise Temperature
20. Algebraic Representation of Noise
21. Additive White Gaussian Noise

1. Introduction

Noise is a general term which is used to describe an unwanted signal which affects a wanted signal. These unwanted signals arise from a variety of sources which may be considered in one of two main categories:-

- *Interference, usually from a human source (man made)*
- *Naturally occurring random noise*

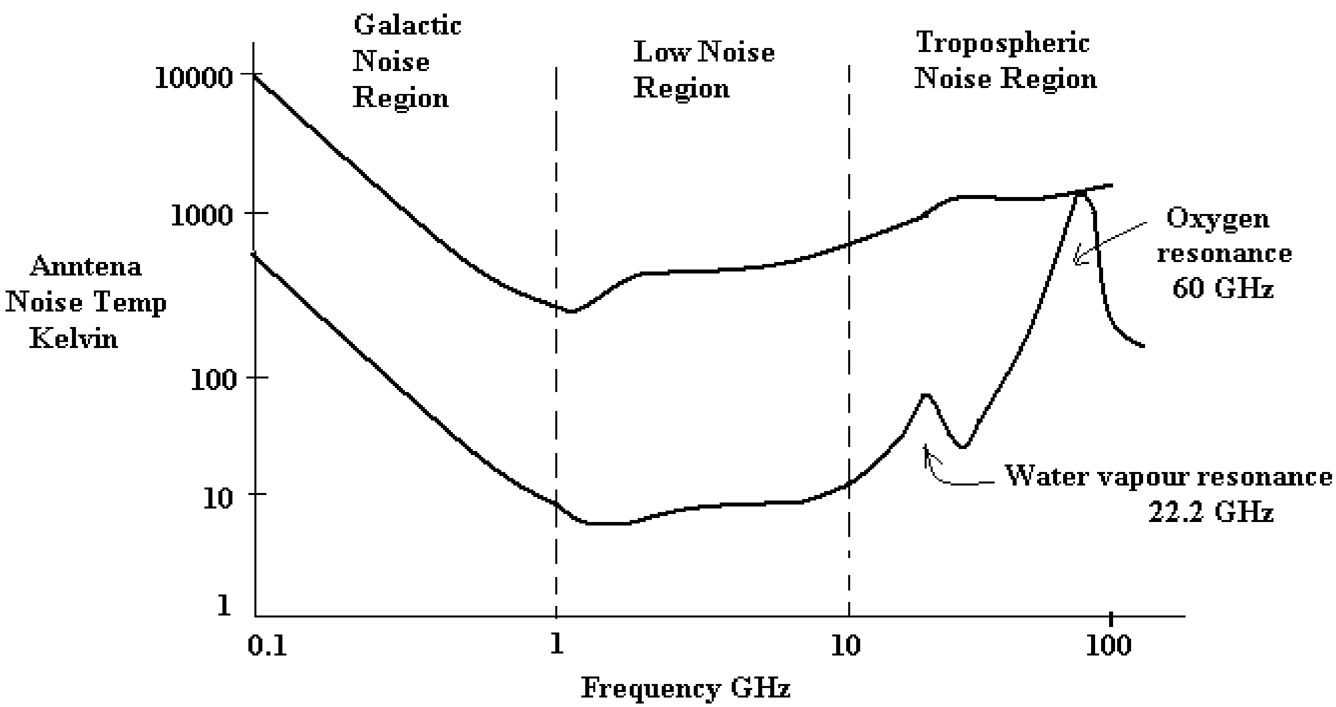
Interference

Interference arises for example, from other communication systems (cross talk), 50 Hz supplies (hum) and harmonics, switched mode power supplies, thyristor circuits, ignition (car spark plugs) motors … etc.

1. Introduction (Cont’d)

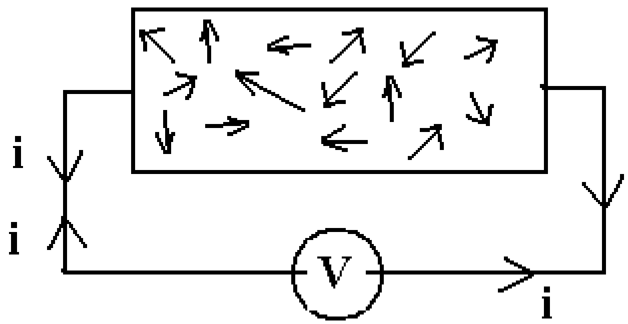
Natural Noise

Naturally occurring external noise sources include atmosphere disturbance (e.g. electric storms, lighting, ionospheric effect etc), so called ‘Sky Noise’ or Cosmic noise which includes noise from galaxy, solar noise and ‘hot spot’ due to oxygen and water vapour resonance in the earth’s atmosphere.



2. Thermal Noise (Johnson Noise)

This type of noise is generated by all resistances (e.g. a resistor, semiconductor, the resistance of a resonant circuit, i.e. the real part of the impedance, cable etc).



Experimental results (by Johnson) and theoretical studies (by Nyquist) give the mean square noise voltage as

$$\bar{V}^2 = 4kTBR \; (volt^2)$$

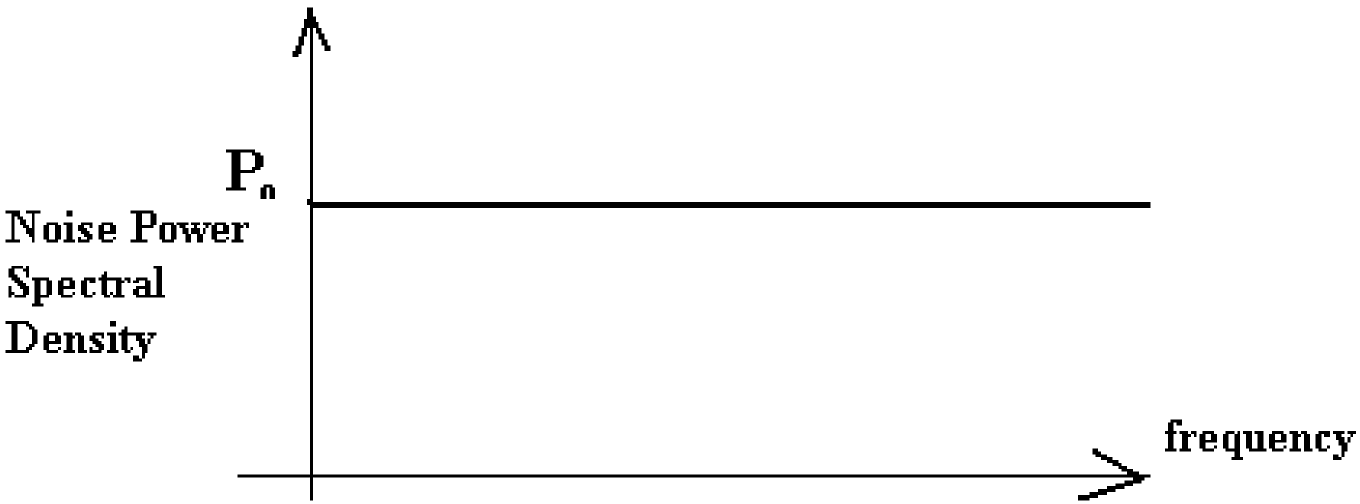
Where k = Boltzmann's constant = 1.38 x 10^{-23} Joules per K
T = absolute temperature
B = bandwidth noise measured in (Hz)
R = resistance (ohms)

2. Thermal Noise (Johnson Noise) (Cont'd)

The law relating noise power, N, to the temperature and bandwidth is

N = k TB watts

Thermal noise is often referred to as 'white noise' because it has a uniform 'spectral density'.



Uniform Noise Power Spectral Density

3. Shot Noise

• Shot noise was originally used to describe noise due to random fluctuations in electron emission from cathodes in vacuum tubes (called shot noise by analogy with lead shot).
• Shot noise also occurs in semiconductors due to the liberation of charge carriers.
• For *pn* junctions the mean square shot noise current is

$$I_n^2 = 2\left(I_{DC} + 2I_o\right)q_e B \quad (amps)^2$$

Where

is the direct current as the *pn* junction (amps)
is the reverse saturation current (amps)
is the electron charge = 1.6 x 10-19 coulombs
B is the effective noise bandwidth (Hz)

• Shot noise is found to have a uniform spectral density as for thermal noise

4. Low Frequency or Flicker Noise

Active devices, integrated circuit, diodes, transistors etc also exhibits a low frequency noise, which is frequency dependent (i.e. non uniform) known as flicker noise or ‘one – over – f’ noise.

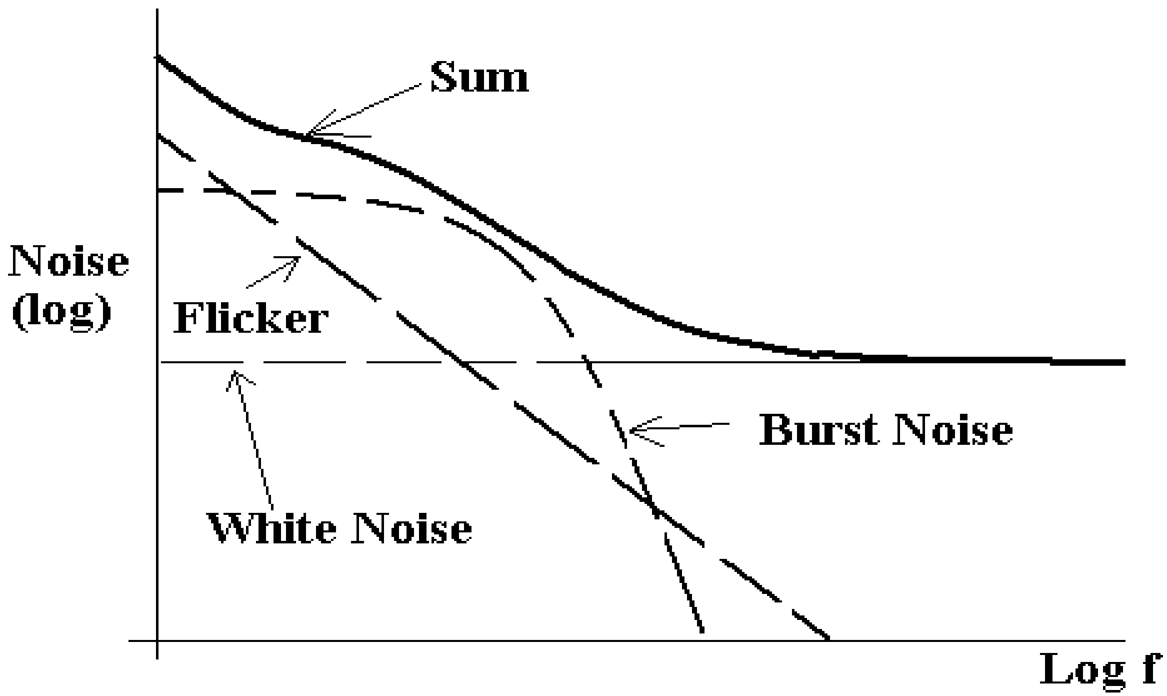
5. Excess Resistor Noise

Thermal noise in resistors does not vary with frequency, as previously noted, by many resistors also generates as additional frequency dependent noise referred to as excess noise.

6. Burst Noise or Popcorn Noise

Some semiconductors also produce burst or popcorn noise with a spectral density which is proportional to $\left(\frac{1}{f}\right)^2$

7. General Comments



For frequencies below a few KHz (low frequency systems), flicker and popcorn noise are the most significant, but these may be ignored at higher frequencies where 'white' noise predominates.

8. Noise Evaluation

The essence of calculations and measurements is to determine the signal power to Noise power ratio, i.e. the (S/N) ratio or (S/N) expression in dB.

$$\left(\frac{S}{N}\right)_{ratio} = \frac{S}{N}$$

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10}\left(\frac{S}{N}\right)$$

$$Also \quad recall \quad that$$

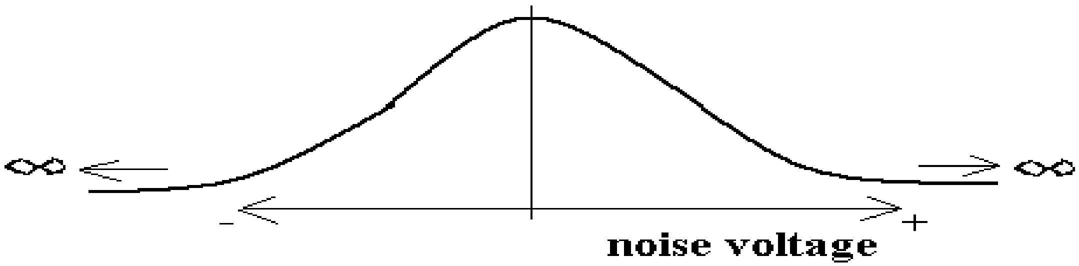
$$S_{dBm} = 10 \log_{10}\left(\frac{S(mW)}{1mW}\right)$$

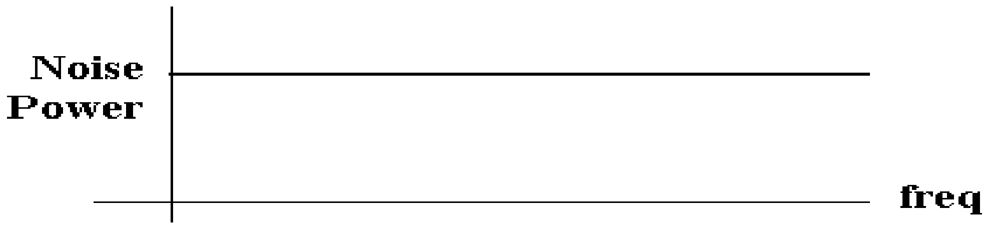
$$and \quad N_{dBm} = 10 \log_{10}\left(\frac{N(mW)}{1mW}\right)$$

$$i.e. \left(\frac{S}{N}\right)_{dB} = 10 \log_{10} S - 10 \log_{10} N$$

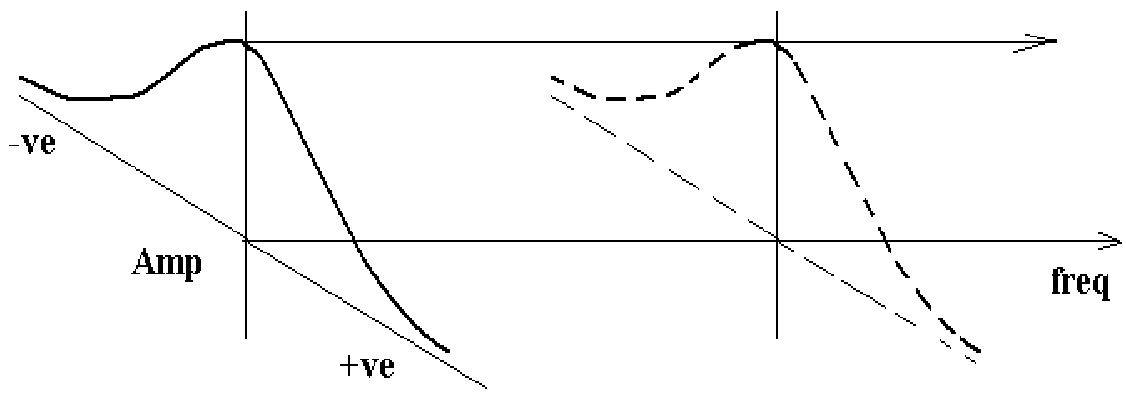
$$\left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm}$$

8. Noise Evaluation (Cont'd)





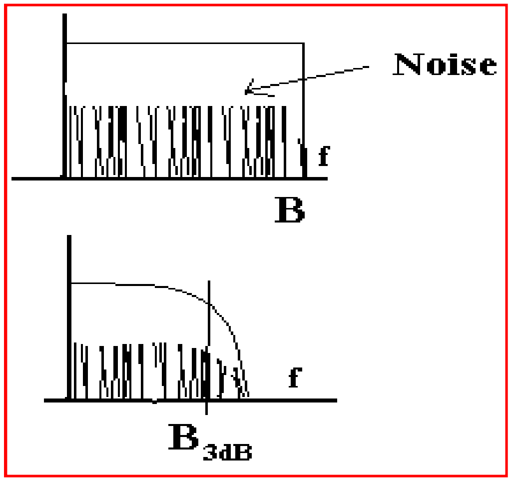
The probability of amplitude of noise at any frequency or in any band of frequencies (e.g. 1 Hz, 10Hz… 100 KHz .etc) is a Gaussian distribution.



8. Noise Evaluation (Cont'd)

Noise may be quantified in terms of noise power spectral density, p_o watts per Hz, from which Noise power N may be expressed as

$$N = p_o B_n \text{ watts}$$



Ideal low pass filter

Bandwidth B Hz = B_n

$N = p_o B_n$ watts

Practical LPF

3 dB bandwidth shown, but noise does not suddenly cease at B_{3dB}

Therefore, $Bn > B_{3dB}$, Bn depends on actual filter.

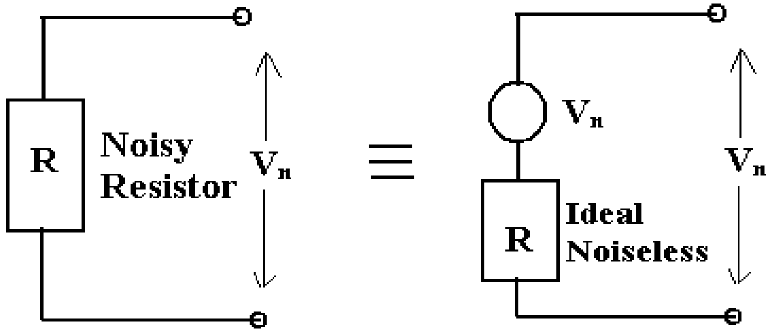
$N = p0\ B_n$

In general the equivalent noise bandwidth is $> B_{3dB}$.

9. Analysis of Noise In Communication Systems

Thermal Noise (Johnson noise)

This thermal noise may be represented by an equivalent circuit as shown below



$$\overline{V^2} = 4kTBR \; (volt^2)$$

(mean square value , power)

then $V_{RMS} = \sqrt{\overline{V^2}} = 2\sqrt{kTBR} = V_n$

i.e. V_n is the RMS noise voltage.

A) System BW = B Hz

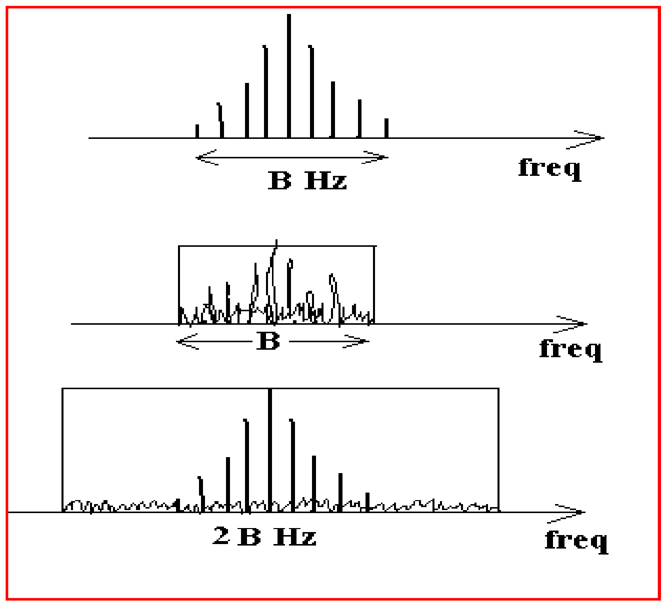
N= Constant B (watts) = KB

B) System BW

N= Constant 2B (watts) = K2B

For A, $\frac{S}{N} = \frac{S}{KB}$

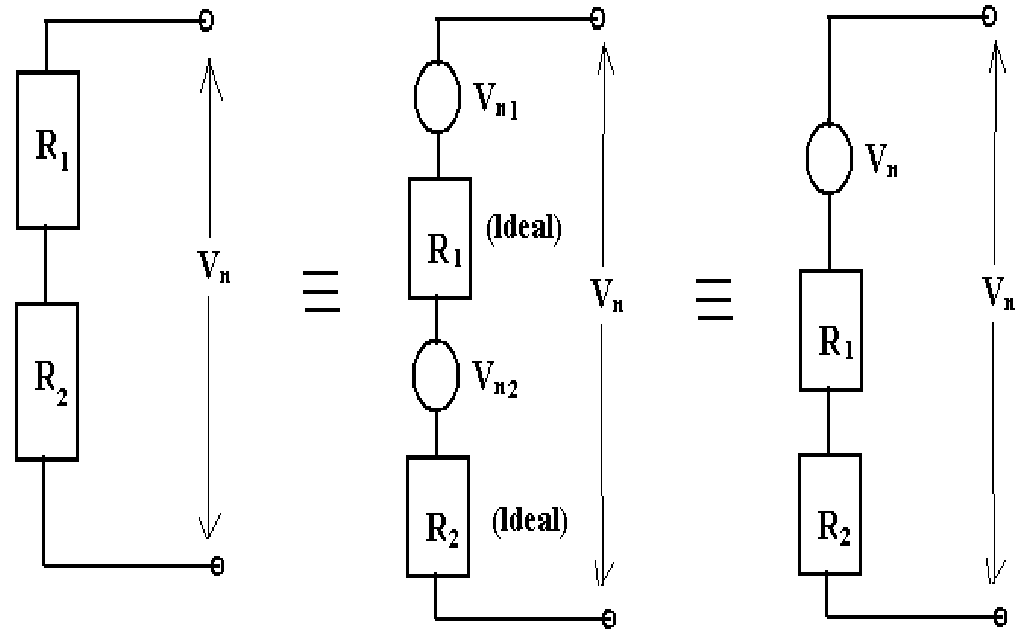
For B, $\frac{S}{N} = \frac{S}{K2B}$



9. Analysis of Noise In Communication Systems (Cont'd)

Resistors in Series

Assume that R_1 at temperature T_1 and R_2 at temperature $T_{2,}$ then



$$\overline{V_n^{\;2}} = \overline{V_{n1}^{\;2}} + \overline{V_{n2}^{\;2}}$$

$$\overline{V_{n1}^{\;2}} = 4kT_1BR_1$$

$$\overline{V_{n2}^{\;2}} = 4kT_2BR_2$$

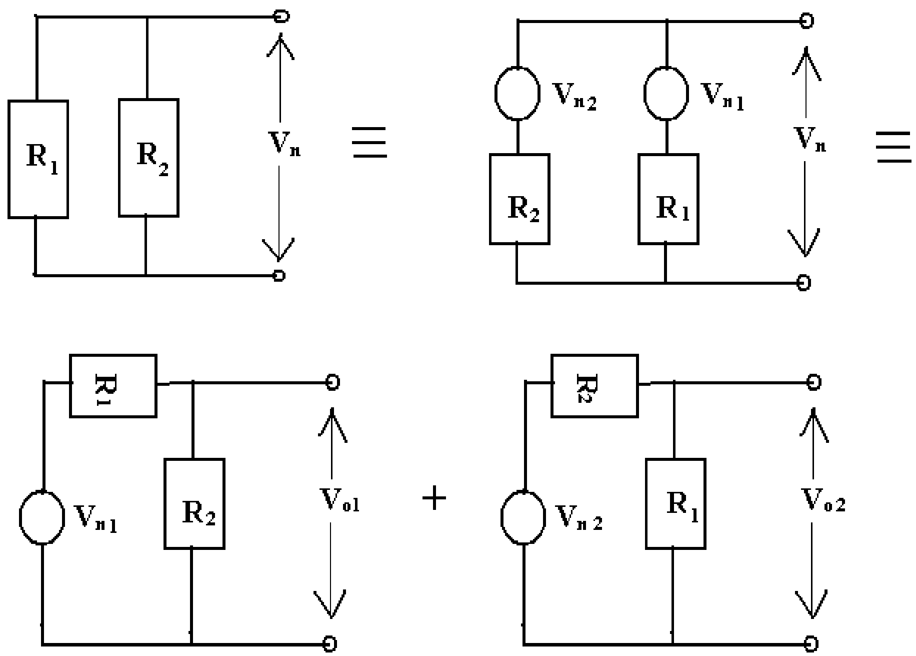
$$\therefore \; \overline{V_n^{\;2}} = 4kB(T_1R_1 + T_2R_2)$$

$$\overline{V_n^{\;2}} = 4kTB(R_1 + R_2)$$

i.e. The resistor in series at same temperature behave as a single resistor

9. Analysis of Noise In Communication Systems (Cont’d)

Resistance in Parallel



$$V_{o1} = V_{n1}\frac{R_2}{R_1 + R_2} \qquad V_{o2} = V_{n2}\frac{R_1}{R_1 + R_2}$$

$$\overline{V_n^{\,2}} = \overline{V_{o1}^{\,2}} + \overline{V_{o2}^{\,2}}$$

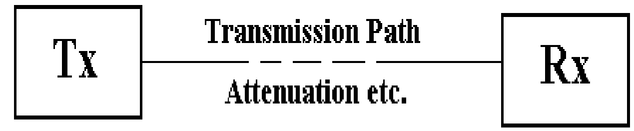
$$\overline{V_n^{\,2}} = \frac{4kB}{(R_1 + R_2)^2}\left[R_2^2\, T_1 R_1 + R_1^2\, T_2 R_2\right] \times \left(\frac{R_1 R_2}{R_1 R_2}\right)$$

$$\overline{V_n^{\,2}} = \frac{4kB\; R_1 R_2\,(T_1 R_1 + T_2 R_2\,)}{(R_1 + R_2)^2}$$

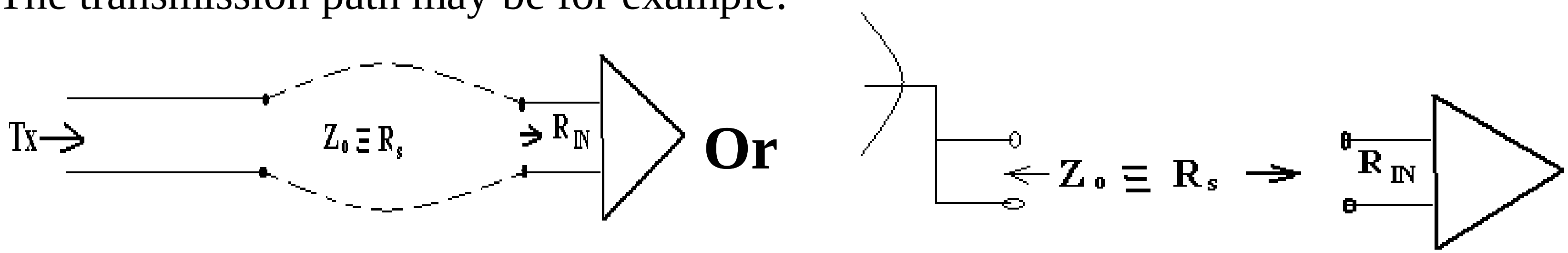
$$\overline{V_n^{2}} = 4kTB\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

10. Matched Communication Systems

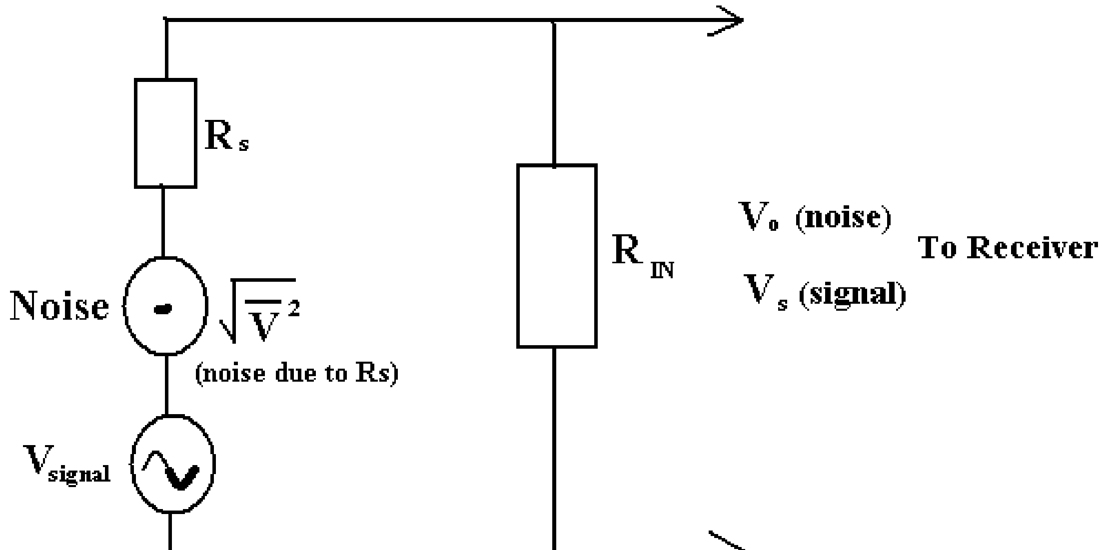
In communication systems we are usually concerned with the noise (i.e. S/N) at the receiver end of the system.



The transmission path may be for example:-



An equivalent circuit, when the line is connected to the receiver is shown below.



10. Matched Communication Systems (Cont'd)

The RMS voltage output, V_o (noise) is

$$V_o(noise) = \sqrt{\overline{v^2}}\left(\frac{R_{IN}}{R_{IN}+R_S}\right)$$

Similarly, the signal voltage output due to V_{signal} at input is

$$Vs(signal) = (V_{signal})\left(\frac{R_{IN}}{R_{IN}+R_s}\right)$$

For maximum power transfer, the input R_{IN} is matched to the source R_S, i.e. R_{IN} $= R_S$ = R (say)

Then $$V_o(noise) = \sqrt{\overline{v^2}}\left(\frac{R}{2R}\right) = \frac{\sqrt{\overline{v^2}}}{2} \quad (RMS\ \ Value)$$

And signal, $V_{S(signal)} = \dfrac{V_{signal}}{2}$

11. Signal to Noise

The signal to noise ratio is given by

$$\frac{S}{N} = \frac{Signal\ Power}{Noise\ Power}$$

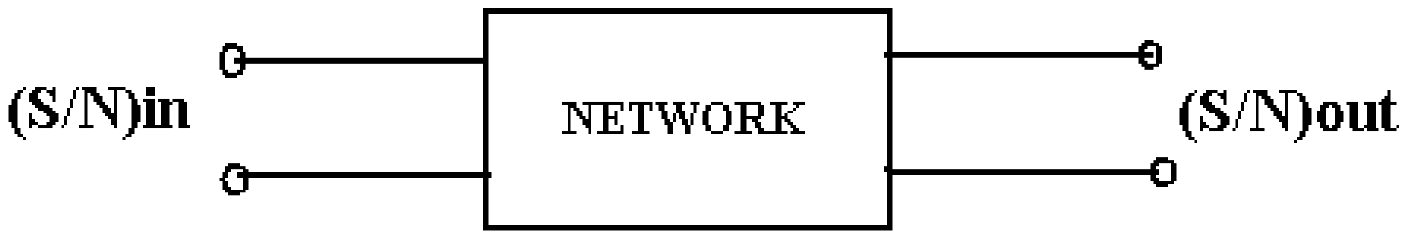
The signal to noise in dB is expressed by

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm}$$ for S and N measured in mW.

12. Noise Factor- Noise Figure

Consider the network shown below,



12. Noise Factor- Noise Figure (Cont'd)

• The amount of noise added by the network is embodied in the Noise Factor F, which is defined by

$$\text{Noise factor F} = \frac{\left(S/N\right)_{IN}}{\left(S/N\right)_{OUT}}$$

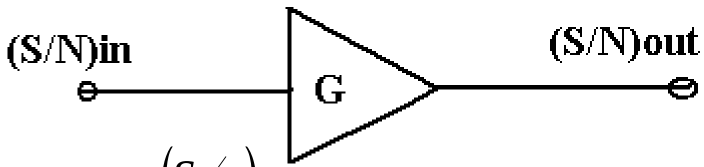
• F equals to 1 for noiseless network and in general F > 1. The noise figure in the noise factor quoted in dB

i.e. Noise Figure F dB = 10 log10 F $\qquad$ F ≥ 0 dB

• The noise figure / factor is the measure of how much a network degrades the (S/N)IN, the lower the value of F, the better the network.

13. Noise Figure – Noise Factor for Active Elements

For active elements with power gain G>1, we have

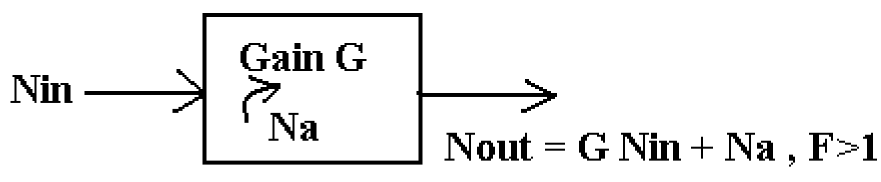


$$F = \frac{\left(S/N\right)_{IN}}{\left(S/N\right)_{OUT}} = \frac{S_{IN}}{N_{IN}}\frac{N_{OUT}}{S_{OUT}} \qquad \text{But} \qquad S_{OUT} = G\,S_{IN}$$

Therefore

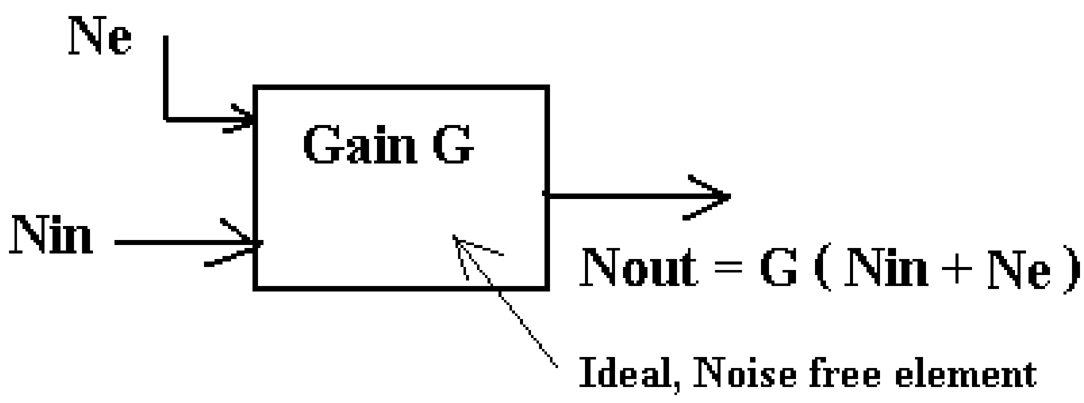
$$F = \frac{S_{IN}}{N_{IN}}\frac{N_{OUT}}{G\,S_{IN}} = \frac{N_{OUT}}{G\,N_{IN}}$$

Since in general F v> 1 , then N_{OUT} is increased by noise due to the active element i.e.



Na represents 'added' noise measured at the output. This added noise may be referred to the input as extra noise, i.e. as equivalent diagram is

13. Noise Figure – Noise Factor for Active Elements (Cont'd)



Ne is extra noise due to active elements referred to the input; the element is thus effectively noiseless.

$$\text{Hence } F = \frac{N_{OUT}}{G\,N_{IN}} = F = \frac{G(N_{IN} + N_e)}{G\,N_{IN}}$$

Rearranging gives,

$$N_e = (F-1)\,N_{IN}$$

14. Noise Temperature

N_{IN} is the 'external' noise from the source i.e. $N_{IN} = kT_S B_n$

T_S is the equivalent noise temperature of the source (usually 290K).

We may also write $N_e = kT_e B_n$, where T_e is the equivalent noise temperature of the element i.e. with noise factor F and with source temperature T_S.

$$\text{i.e. } kT_e B_n = (F\text{-}1)\, kT_S B_n$$

$$\text{or } T_e = (F\text{-}1) T_S$$

15. Noise Figure – Noise Factor for Passive Elements

Since $F = \frac{S_{IN}}{N_{IN}} \frac{N_{OUT}}{S_{OUT}}$ and $N_{OUT} = N_{IN}$.

$$F = \frac{S_{IN}}{G S_{IN}} = \frac{1}{G}$$

If we let L denote the insertion loss (ratio) of the network i.e. insertion loss

$$L_{dB} = 10 \log L$$

Then

$$L = \frac{1}{G}$$ and hence for passive network

$$F = L$$

Also, since $T_e = (F\text{-}1) T_S$

Then for passive network

$$T_e = (L\text{-}1) T_S$$

Where T_e is the equivalent noise temperature of a passive device referred to its input.

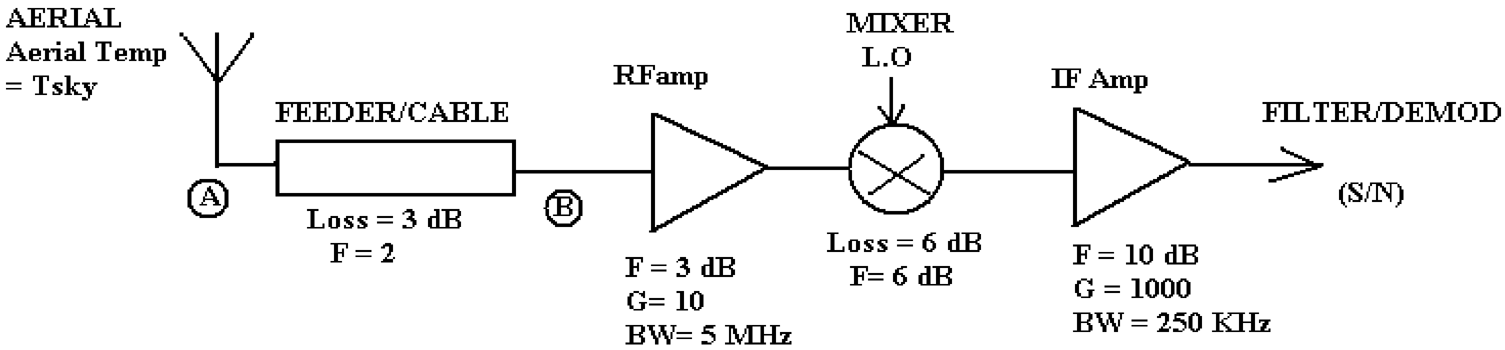
16. Review of Noise Factor – Noise Figure –Temperature

Typical values of noise temperature, noise figure and gain for various amplifiers and attenuators are given below:

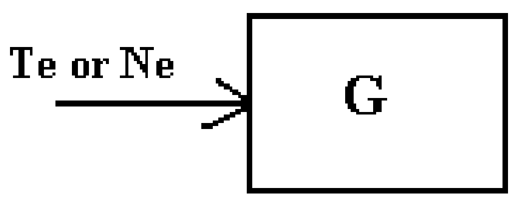
Device	**Frequency**	T_e **(K)**	F_{dB} **(dB)**	**Gain (dB)**
Maser Amplifier	9 GHz	4	0.06	20
Ga As Fet amp	9 GHz	330	303	6
Ga As Fet amp	1 GHz	110	1.4	12
Silicon Transistor	400 MHz	420	3.9	13
L C Amp	10 MHz	1160	7.0	50
Type N cable	1 GHz		2.0	2.0

17. Cascaded Network

A receiver systems usually consists of a number of passive or active elements connected in series. A typical receiver block diagram is shown below, with example



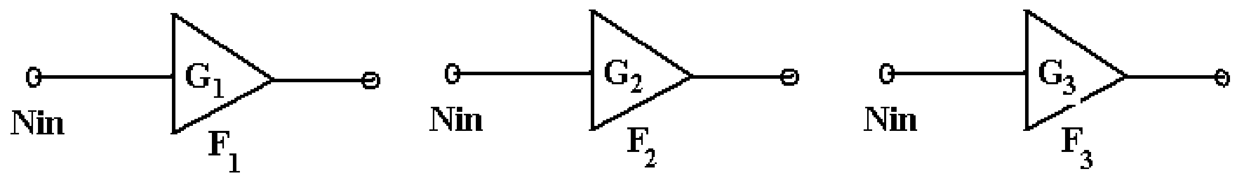
In order to determine the (S/N) at the input, the overall receiver noise figure or noise temperature must be determined. In order to do this all the noise must be referred to the same point in the receiver, for example to A, the feeder input or B, the input to the first amplifier.



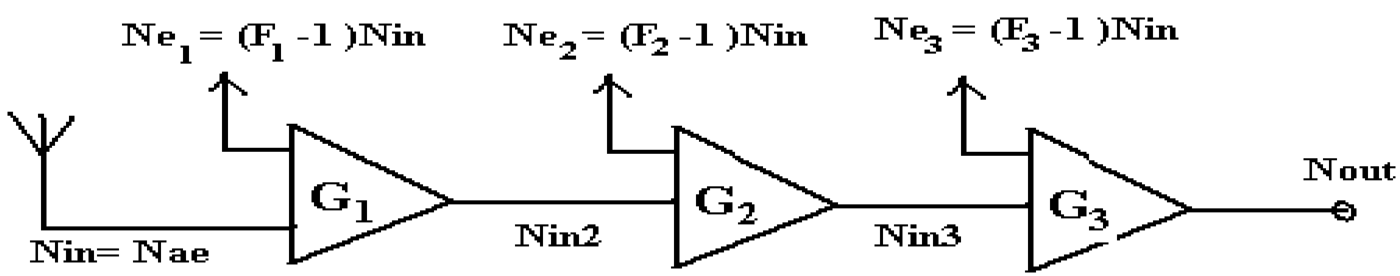
T_e or N_e is the noise referred to the input.

18. System Noise Figure

Assume that a system comprises the elements shown below,



Assume that these are now cascaded and connected to an aerial at the input, with $N_{IN} = N_{ae}$ from the aerial.



Now,
$$N_{OUT} = G_3\left(N_{IN3} + N_{e3}\right)$$
$$= G_3\left(N_{IN3} + \left(F_3 - 1\right)N_{IN}\right)$$

Since
$$N_{IN3} = G_2\left(N_{IN2} + N_{e2}\right) = G_2\left(N_{IN2} + \left(F_2 - 1\right)N_{IN}\right)$$

similarly
$$N_{IN2} = G_1\left(N_{ae} + \left(F_1 - 1\right)N_{IN}\right)$$

18. System Noise Figure (Cont'd)

$$N_{OUT} = G_3\left[G_2\left[G_1 N_{ae} + G_1\left(F_1 - 1\right)N_{IN}\right] + G_2\left(F_2 - 1\right)N_{IN}\right] + G_3\left(F_3 - 1\right)N_{IN}$$

The overall system Noise Factor is

$$F_{sys} = \frac{N_{OUT}}{GN_{IN}} = \frac{N_{OUT}}{G_1 G_2 G_3 N_{ae}}$$
$$= 1 + \left(F_1 - 1\right)\frac{N_{IN}}{N_{ae}} + \frac{\left(F_2 - 1\right)}{G_1}\frac{N_{IN}}{N_{ae}} + \frac{\left(F_3 - 1\right)}{G_1 G_2}\frac{N_{IN}}{N_{ae}}$$

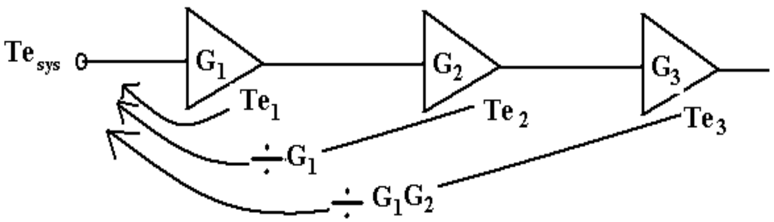
If we assume N_{ae} is $\approx N_{IN}$, i.e. we would measure and specify F_{sys} under similar conditions as F_1, F_2 etc (i.e. at 290 K), then for n elements in cascade.

$$F_{sys} = F_1 + \frac{\left(F_2 - 1\right)}{G_1} + \frac{\left(F_3 - 1\right)}{G_1 G_2} + \frac{\left(F_4 - 1\right)}{G_1 G_2 G_3} + \ldots\ldots\ldots + \frac{\left(F_n - 1\right)}{G_1 G_2 \ldots\ldots\ldots G_{n-1}}$$

The equation is called FRIIS Formula.

19. System Noise Temperature

Since $T_e = (L-1)T_S$, i.e. $F = 1 + \dfrac{T_e}{T_s}$



Then

$$F_{sys} = 1 + \frac{T_{e\,sys}}{T_s} \qquad \begin{cases} \text{where } T_{e\,sys} \text{ is the equivalent Noise temperature of the system} \\ \text{and } T_s \text{ is the noise temperature of the source} \end{cases}$$

and

$$\left(1 + \frac{T_{e\,sys}}{T_s}\right) = \left(1 + \frac{T_{e1}}{T_s}\right) + \frac{\left(1 + \frac{T_{e2}}{T_s} - 1\right)}{G_1} + \ldots etc$$

$$i.e.\ \ from\ F_{sys} = F_1 + \frac{(F_2 - 1)}{G_1} + \ldots\ldots etc$$

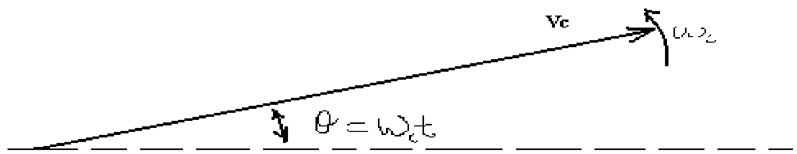
which gives

$$T_{e\,sys} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \frac{T_{e4}}{G_1G_2G_3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

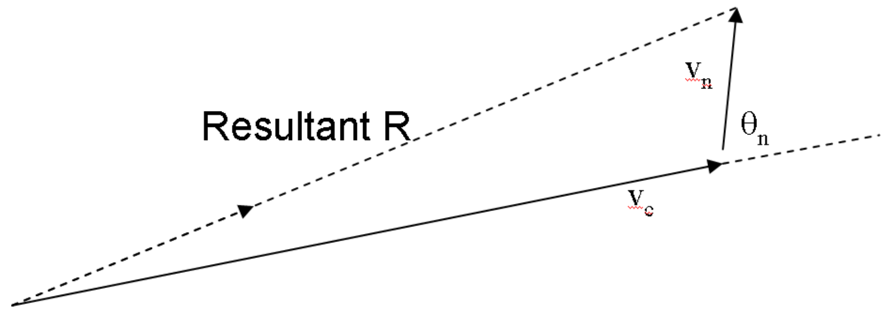
20. Algebraic Representation of Noise

Phasor Representation of Signal and Noise

The general carrier signal VcCosWct may be represented as a phasor at any instant in time as shown below:



If we now consider a carrier with a noise voltage with "peak" value superimposed we may represents this as:



Both Vn and θ_n are random variables, the above phasor diagram represents a snapshot at some instant in time.

Chapter 5. Noise in CW Modulation System

Noise in CW Modulation System

- 5.1 Introduction
  - \- Receiver Noise (Channel Noise) :
  - additive, White, and Gaussian 으로 가정

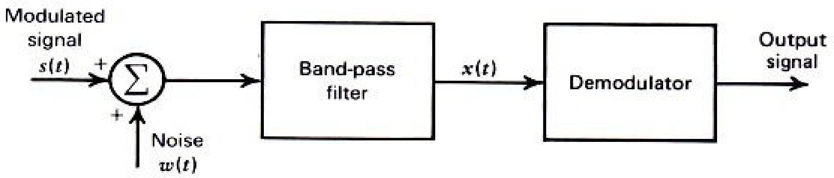
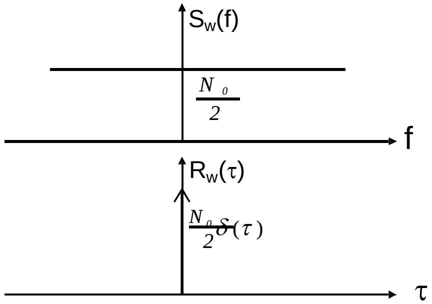
- 5.2 Receiver Model
- 1. RX Model
- 

Figure 5.1 Noisy receiver model.

\- w(t): additive, white, and Gaussian, power spectral density $= \frac{N_0}{2}$



- $N_0 = KT_e$ where K = Boltzmann's constant
- T_e = equivalent noise Temp.
- Average noise power per unit bandwidth

- - Band Pass Filter (Ideal

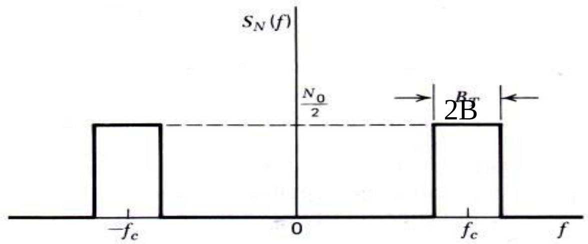
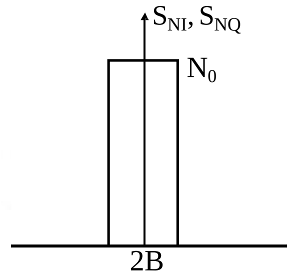


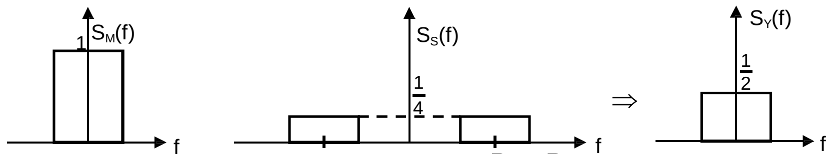
Figure 5.2 Idealized characteristic of band-pass filtered noise.



- w(t) n(t)
-
- - filtered noise as narrow-band noise
- $n(t) = n_I(t)\cos(2\pi f_C t) - n_Q(t)\sin(2\pi f_C t)$
  - where $n_I(t)$ is inphase, $n_Q(t)$ is quadrature component
- - $\therefore$ filtered signal x(t)
- $x(t) = s(t) + n(t)$
-
- - Average Noise Power = $N_0 B_T$
- $$-\ (SNR)_I = \frac{\text{average power of the modulated signal } s(t)}{\text{average power of the filtered noise } n(t)}$$

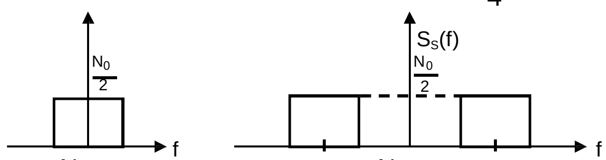
$$-\ (SNR)_O = \left.\frac{\text{average power of the demodulated message signal}}{\text{average power of the noise}}\right|_{\text{receiver output}}$$

- DSB



$m(t) \to P$ $\quad$ $m(t)\cos(2\pi f_C t) \to 2\times\left(\frac{P}{4}\right) = \frac{P}{2}$

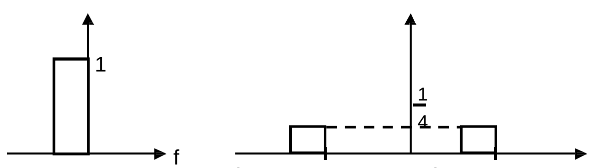
- $s(t) = m(t)\cos(2\pi f_C t + \Theta) \to S_S(f) = \frac{1}{4}\left[S_M(f - f_C) + S_M(f + f_C)\right]$



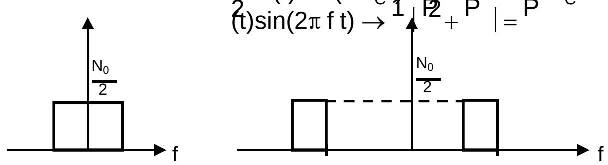
$2W\frac{N_0}{2} = WN_0$ $\quad$ $4W\frac{N_0}{2} = 2WN_0$

$\frac{1}{2}m(t) + \frac{1}{2}n_I(t)$

- SSB



$\frac{1}{2}m(t)\cos(2\pi f_C t) + \frac{1}{2}\hat{m}(t)\sin(2\pi f_C t) \to \frac{1}{4}\left(\frac{P}{2} + \frac{P}{2}\right) = \frac{P}{4}$

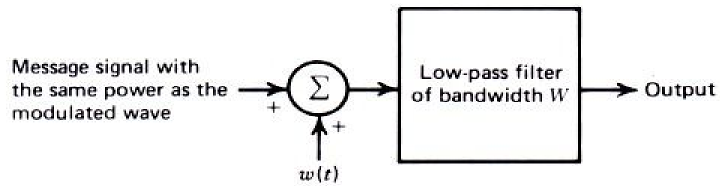


$2W\frac{N_0}{2} = WN_0$ $\quad$ $2W\frac{N_0}{2} = WN_0$

$\frac{1}{4}m(t) + \frac{1}{2}n_I(t)\cos(\pi Wt) + \frac{1}{2}n_Q(t)\sin(\pi Wt)$

- - s(t) by each system has the same average power
- - noise w(t) has the same average power measured in the message
- BW=W

- 1) Channel SNR



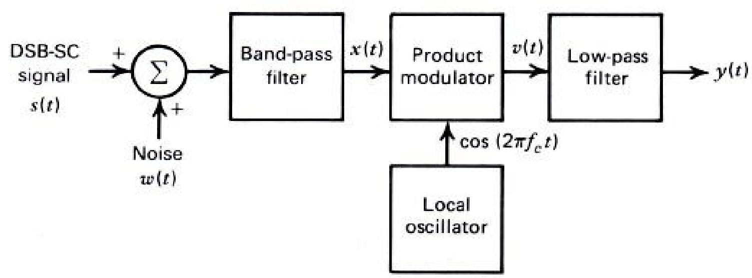
$$(SNR)_C = \left.\frac{\text{average power of the modulated signal}}{\text{average power of the noise in the message BW}}\right|_{\text{at receiver input}}$$

- 2) $$\text{Figure of merit} = \frac{(SNR)_O}{(SNR)_C}$$

- Noise in DSB-SC

R

-



-

2. (SNR) C

- $s(t) = CA_C \cos(2\pi f_C t)m(t)$

whe re C : scaling factor Power spectral density of m(t) : $S_M(f)$ W : message bandwidth

- Average signal power

$$P = \int_{-W}^{W} S_M(f)\, df$$

- Average power of s(t) $= \dfrac{C^2 A_C^2 P}{2}$

- Average noise power $= 2W \times \dfrac{N_0}{2} = WN_0$ (baseband)

- $(SNR)_{C,DSB} = \dfrac{C^2 A_C^2 P}{2WN_0}$

- $x(t) = s(t) + n(t)$

$= CA_C \cos(2\pi f_C t)m(t) + n_I(t)\cos(2\pi f_C t) - n_Q(t)\sin(2\pi f_C t)$

- $v(t) = x(t)\cos(2\pi f_C t)$

$= \frac{1}{2}CA_C m(t) + \frac{1}{2}n_I(t) + \frac{1}{2}\left[CA_C m(t) + n_I(t)\right]\cos(4\pi f_C t) - \frac{1}{2}A_C n_Q(t)\sin(4\pi f_C t)$

$\therefore y(t) = \frac{1}{2}CA_C m(t) + \frac{1}{2}n_I(t)$

- Average signal power $= \dfrac{C^2 A_C^2 P}{4}$

- Average noise power $= \dfrac{1}{4}(2W)N_0 = \dfrac{1}{2} W N_0$ (passband)

  $\because$ Power$(n_I(t))$ = Power of bandpass filtered noise $n(t) = 2WN_0$

- $\therefore (SNR)_O = \dfrac{C^2 A_C^2 P/4}{WN_0/2} = \dfrac{C^2 A_C^2 P}{2WN_0}$

  $\therefore$ Figure of merit

$$\left.\frac{(SNR)_O}{(SNR)_C}\right|_{DSB-SC} = 1$$

Noise in SSB Receivers

- SSB Modulated wave

$$s(t) = \frac{1}{2} C A_C \cos(2\pi f_C t) m(t) + \frac{1}{2} C A_C \sin(2\pi f_C t) \hat{m}(t)$$

- $m(t)$ and $\hat{m}(t)$ are othogonal, $E[m(t)\hat{m}(t)] = 0$

  $\Rightarrow m(t)$ and $\hat{m}(t)$ are uncorrelated

  $\Rightarrow$ their power spectral densities are additive

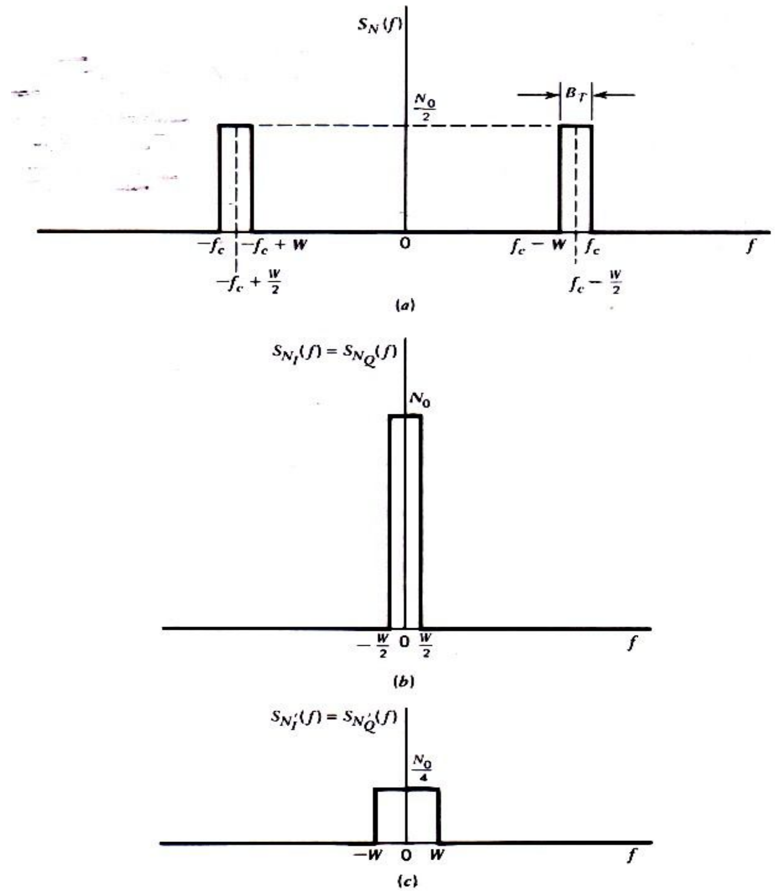
- $m(t)$ and $\hat{m}(t)$ has the same power spectral density

- Message power $= \dfrac{C^2 A_C^2}{4}\dfrac{P}{2} + \dfrac{C^2 A_C^2}{4}\dfrac{P}{2} = \dfrac{C^2 A_C^2 P}{4}$

  (half of DSB)

  - Average noise power $= WN_0$ ($\because$ message BW 안의 Noise) (baseband)

$$(SNR)_{C,SSB} = \frac{C^2 A_C^2 P}{4WN_0}$$



$$n(t) = n_I(t)\cos\left(2\pi(f_C - \frac{W}{2})t\right) - n_Q(t)\sin\left(2\pi(f_C - \frac{W}{2})t\right)$$

- Combined output

$$y(t) = \frac{1}{4} CA_C m(t) + \frac{1}{2} n_I(t)\cos(\pi Wt) + \frac{1}{2} n_Q(t)\sin(\pi Wt)$$

- Average signal power $= \frac{1}{16} C^2 A_C^2 P$

- Average noise power $= \frac{1}{4}\frac{WN_0}{2} + \frac{1}{4}\frac{WN_0}{2} = \frac{1}{4} WN_0$ (passband)

- $(SNR)_{O,SSB} = \frac{C^2 A_C^2 P}{4WN_0}$

- Figure of merit $\left.\frac{(SNR)_O}{(SNR)_C}\right|_{SSB} = 1$ same as DSB - SC

5.4 Noise in AM Receiver

- AM signal

$s(t) = A_C[1 + k_a m(t)]\cos(2\pi f_C t)$

- Average signal power $= A_C^2(1 + k_a^2 P)/2$
- Average noise power $= WN_0 \leftarrow (2W \times \frac{N_0}{2})$

$$(SNR)_{C,AM} = \frac{A_C^2(1 + k_a^2 P)}{2WN_0}$$

- Filtered signal $x(t) = s(t) + n(t)$

$= [A_C + A_C k_a m(t) + n_I(t)]\cos(2\pi f_C t) - n_Q(t)\sin(2\pi f_C t)$

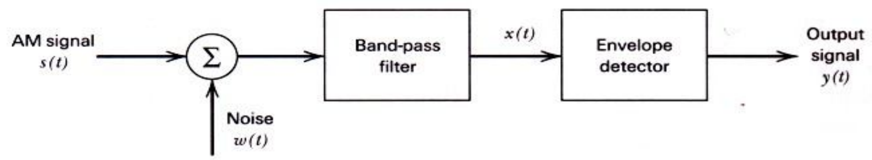


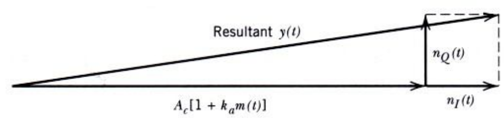
Figure 5.6 Noisy model of AM receiver.

$y(t) = \text{envelop of } x(t)$

$$= \left\{ [A_C + A_C k_a m(t) + n_I(t)]^2 + n_Q^2(t) \right\}^{\frac{1}{2}}$$

Assume $A_C[1 + k_a m(t)] >> n_I(t), n_Q(t)$

$y(t) \cong A_C + A_C k_a m(t) + n_I(t)$



- $(SNR)_{O,AM} = \dfrac{A_C^2 k_a^2 P}{2W N_0}$

조건 $\begin{cases} \text{Avg carrier power} > \text{Avg noise power} \\ k_a \le 1 \end{cases}$

- Figure of merit $\left.\dfrac{(SNR)_O}{(SNR)_C}\right|_{AM} \cong \dfrac{k_a^2 P}{1 + k_a^2 P} < 1$

- ex1) Single-Tone Modulation

$m(t) = A_m \cos(2\pi f_m t) \rightarrow P = \dfrac{A_m^2}{2}$

$s(t) = A_C[1 + \mu \cos(2\pi f_m t)]\cos(2\pi f_C t)$ where $\mu = k_a A_m$

$$\left.\frac{(SNR)_O}{(SNR)_C}\right|_{AM} = \frac{\frac{1}{2}k_a^2 A_m^2}{1 + \frac{1}{2}k_a^2 A_m^2} = \frac{\mu^2}{2 + \mu^2}$$

if $\mu = 1$, $\text{F.O.M} = \dfrac{1}{3}(\max)$

Threshold Effect

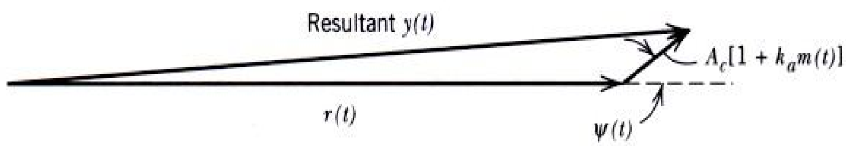
- Carrier-to-noise < 1
- narrow-band noise n(t)

$n(t) = r(t)\cos[2\pi f_C (t) + \psi(t)]$

where $r(t)$ is envelope, $\psi(t)$ is phase

$x(t) = s(t) + n(t)$

$= A_C [1 + k_a m(t)]\cos(2\pi f_C t) + r(t)\cos(2\pi f_C t + \psi(t))$



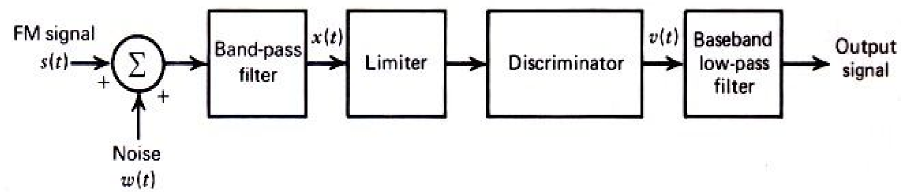
$$y(t) \cong r(t) + A_C [1 + k_a m(t)]\cos[\psi(t)]$$

$\cong r(t) + A_C\cos[\psi(t)] + A_C k_a m(t)\cos[\psi(t)]$ where $\psi(t)$ is uniformly distributed over $[0, 2\pi]$

$\Rightarrow$ complete loss of information

- Threshold Effect : loss of message in an envelope detector that
- operates at a low CNR.
-

Noise in FM Receivers



- w(t) : zero mean white Gaussian noise with psd = $N_0/2$
- s(t) : carrier $=f_c$, BW $= B_T$ 즉 $(f_C \pm B_T/2)$

- - BPF : $[f_C - B_T/2 \sim f_C + B_T/2]$
- - Amplitude limiter : remove amplitude variations by clipping and BPF
- - Discriminator
  - slope network or differentiator : varies linearly with frequency
  - envelope detector
- - Baseband LPF :

$$s(t) = A_C \cos[2\pi f_C t + 2\pi k_f \int_0^t m(t)dt]$$

$$\phi(t) = 2\pi k_f \int_0^t m(t)dt$$

$$s(t) = A_C \cos[2\pi f_C t + \phi(t)]$$

- - FM signal

$$n(t) = n_I(t)\cos(2\pi f_C t) + n_Q(t)\sin(2\pi f_C t)$$

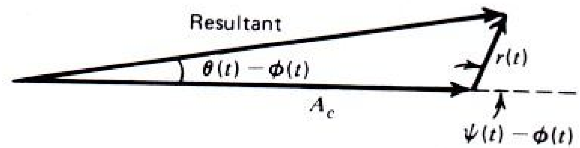
$$= r(t)\cos[2\pi f_C t + \psi(t)]$$

- - Filtered noise n(t)

where $\begin{cases} r(t) = \sqrt{(n_I(t))^2 + (n_Q(t))^2} \\ \psi(t) = \tan^{-1}\left[\dfrac{n_Q(t)}{n_I(t)}\right] \end{cases}$

$$\therefore\ x(t) = s(t) + n(t)$$

$$= A_C \cos[2\pi f_C t + \phi(t)] + r(t)\cos[2\pi f_C t + \psi(t)]$$

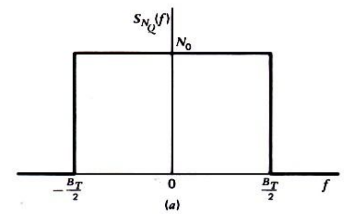


$$\text{where } \Theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin[\psi(t) - \phi(t)]}{A_C + r(t)\cos[\psi(t) - \phi(t)]}\right\}$$

Assume $A_C \gg r(t)$

$$\Theta(t) \cong \phi(t) + \frac{r(t)}{A_C}\sin[\psi(t) - \phi(t)]$$

$$\cong 2\pi k_f \int_0^t m(t)dt + \frac{r(t)}{A_C}\sin[\psi(t) - \phi(t)]$$



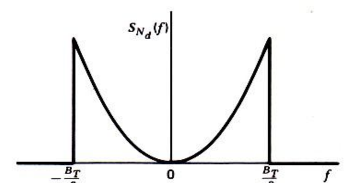
$$v(t) = \frac{1}{2\pi}\frac{d\Theta(t)}{dt} \cong k_f m(t) + n_d(t)$$

where

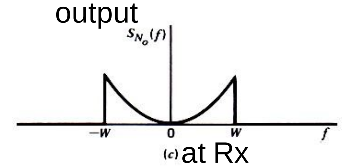
$$n_d(t) = \frac{1}{2\pi A_C}\frac{d}{dt}\{r(t)\sin[\psi(t) - \phi(t)]\}$$

$$\cong \frac{1}{2\pi A_C}\frac{d}{dt}\{r(t)\sin[\psi(t)]\}$$

$$\therefore\ n_d(t) = \frac{1}{2\pi A_C}\frac{dn_Q(t)}{dt}$$



at discriminator output



at Rx output

Pre-emphasis and de-emphasis in FM

- P.S.D. of noise at FM Rx output

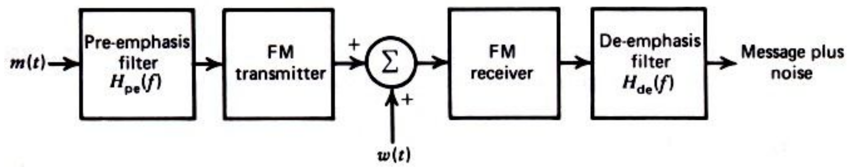
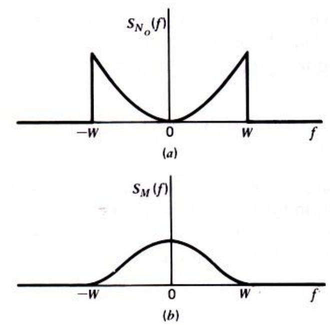


Figure 5.17 Use of pre-emphasis and de-emphasis in an FM system.



signa
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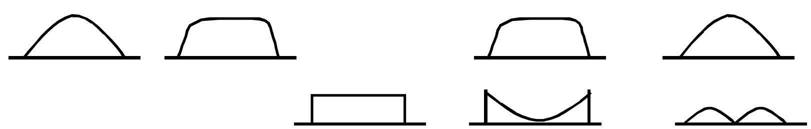
noise



$$H_{de}(f) = \frac{1}{H_{pe}(f)}, \qquad -W \le f \le W$$

P.S.D of noise $n_d(t)$ at the discriminator output

$$S_{Nd}(f) = \begin{cases} \dfrac{N_0 f^2}{A_C^2} & |f| \le \dfrac{B_T}{2} \\ 0 & \text{otherwise} \end{cases}$$